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COMPREHENSIVE ANALYSIS OF  
HELICOPTERS WITH BEARINGLESS ROTORS

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## ABSTRACT

A modified Galerkin's method is developed to analyze the dynamic problems of multiple-load-path bearingless rotor blades. The development and selection of functions are quite parallel to CAMRAD procedures. This greatly facilitates the implementation of the method into the CAMRAD program. A software is developed implementing the modified Galerkin's method to determine free vibration characteristics of multiple-load-path rotor blades undergoing coupled flapwise bending, chordwise bending, twisting, and extensional motions. Results are in the process of being obtained by debugging the software.

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## NOMENCLATURE

A	Area of cross-section
e	Mass centroid offset from elastic axis
E	Young's modulus
G	Shear modulus
$I_R$	Reference moment of inertia
$I_1, I_2$	Bending moments of inertia about major and minor neutral axes
$k_m$	Mass polar radius of gyration of blade cross section about elastic axis
$k_{m1}, k_{m2}$	Mass radii of gyration about major neutral axis and about an axis perpendicular to chord
$\ell$	Length of the load paths
m	Mass per unit length
$m_\ell$	Reference mass
$M_x$	Twisting moment about x-axis
$M_y, M_z$	Bending moments about y and z axes, respectively
n	Number of load path
N	Number of selected functions
R	Radius of the rotor
T	Tension
u, v, w	Elastic displacements in the x, y, z directions, respectively
$V_x$	Axial force
$V_y, V_z$	Shear forces along y and z directions, respectively
x, y, z	Mutually perpendicular axis system with x - along the blade elastic axis y - towards the leading edge

$\beta$	Pretwist angle
$\phi$	Elastic twist about the elastic axis
$\omega$	Frequencies of vibration
$\Omega$	Blade rotational speed

## 1. INTRODUCTION

The bearingless rotorcraft offers reduced weight, less complexity, and superior flying qualities. Bearingless rotor technology was successfully applied by Sikorsky in the tail rotors of the Blackhawk and S-76 helicopters (Ref. 1). Boeing Vertol built the first successful bearingless main rotor (BMR) and flew it in 1978 (Refs. 2 and 3). During a recent study for concept definition of an Integrated Technology Rotor/Flight Research Rotor (ITR/FRR), thirty-three hub concepts were proposed and twenty-one of these were bearingless designs (Ref. 4). This suggests that the next generation of rotorcraft will most likely be equipped with bearingless rotors. Boeing Vertol is currently designing and advanced bearingless rotor hub for MBB-105 blade (ABRS Project).

All practical designs of bearingless rotors include multiple load paths and one that was flight tested by Boeing Vertol has three load paths. The existence of multiple load paths change the dynamic behavior of the helicopter significantly, and also provides a variety of options to the designer. Therefore, it is important to develop a capability to analyze the helicopters with multiple-load-path bearingless hubs. CAMRAD (Ref. 5) is a comprehensive rotorcraft analysis program developed at NASA-Ames. This program utilizes recently developed technology to analyze the following problems:

1. Trim solution
2. Performance, loads and noise
3. Stability derivatives and handling qualities



4. Aeroelastic stability
5. Vibration and gust response

CAMRAD does not model multiple load paths and many users need this capability in CAMRAD.

The modification of CAMRAD program to include the multiple load path is undertaken under the present program. This modification is a major effort requiring several years of concentrated study. The modification is undertaken basically in two major phases:

Phase 1: Extension and validation of CAMRAD procedures to include multiple load paths.

Phase 2: Incorporation of procedures developed under phase 1 into the CAMRAD code under consultation with Dr. Wayne Johnson.

The first two years will be spent for Phase 1 studies. As a part of this phase, a systematic study is undertaken to extend the CAMRAD technologies. CAMRAD uses Galerkin's method (Ref. 5) following reference 6. The limitations of this method as exists in CAMRAD are:

1. Single-load-path modelling can only be done
2. Axial degree-of-freedom is not included in the analysis, and this degree-of-freedom is important for multiple-load-path modelling
3. There is no coupling between bending and torsion in the blade modes

Therefore, a modified Galerkin's method as employed in CAMRAD following reference 6 is extended for bearingless multiple-load-path blade. A computer program is developed to validate the formulation. The

details of the formulation and the listing of the computer program are included in this status report. The results are in the process of being obtained.

## 2. BASIC EQUATIONS

The linear, homogeneous, undamped equations of motion for simple harmonic free vibrations with frequency  $\omega$  can be written as (Ref. 7)

$$-(EAu')' - \Omega^2_{mu} - \omega^2_{mu} = 0 \quad (1)$$

$$\begin{aligned} & -(Tv')' + \{(EI_1 \sin^2 \beta + EI_2 \cos^2 \beta)v'' \\ & + (EI_2 - EI_1) \cos \beta \sin \beta w''\}'' - \Omega^2_{mv} - \omega^2_{mv} \\ & + \Omega^2_{mesin\beta\phi} + \omega^2_{mesin\beta\phi} + (\Omega^2_{mexsin\beta\phi})' = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} & -(Tw')' + \{(EI_1 \cos^2 \beta + EI_2 \sin^2 \beta)w'' \\ & + (EI_2 - EI_1) \cos \beta \sin \beta v''\}'' - \omega^2_{mw} - \omega^2_{mecos\beta\phi} \\ & - (\Omega^2_{mexcos\beta\phi})' = 0 \end{aligned} \quad (3)$$

$$\begin{aligned} & -(GJ\phi')' - \omega^2_{mk_m^2} + \Omega^2_m(k_{m_2}^2 - k_{m_1}^2) \cos 2\beta\phi \\ & - \Omega^2_{mex}(-\sin \beta v' + \cos \beta w') \\ & + \Omega^2_{mesin\beta v} + \omega^2_{mesin\beta v} - \omega^2_{mecos\beta w} = 0 \end{aligned} \quad (4)$$

$$T = \int_x^R \Omega^2_m x dx \quad (5)$$

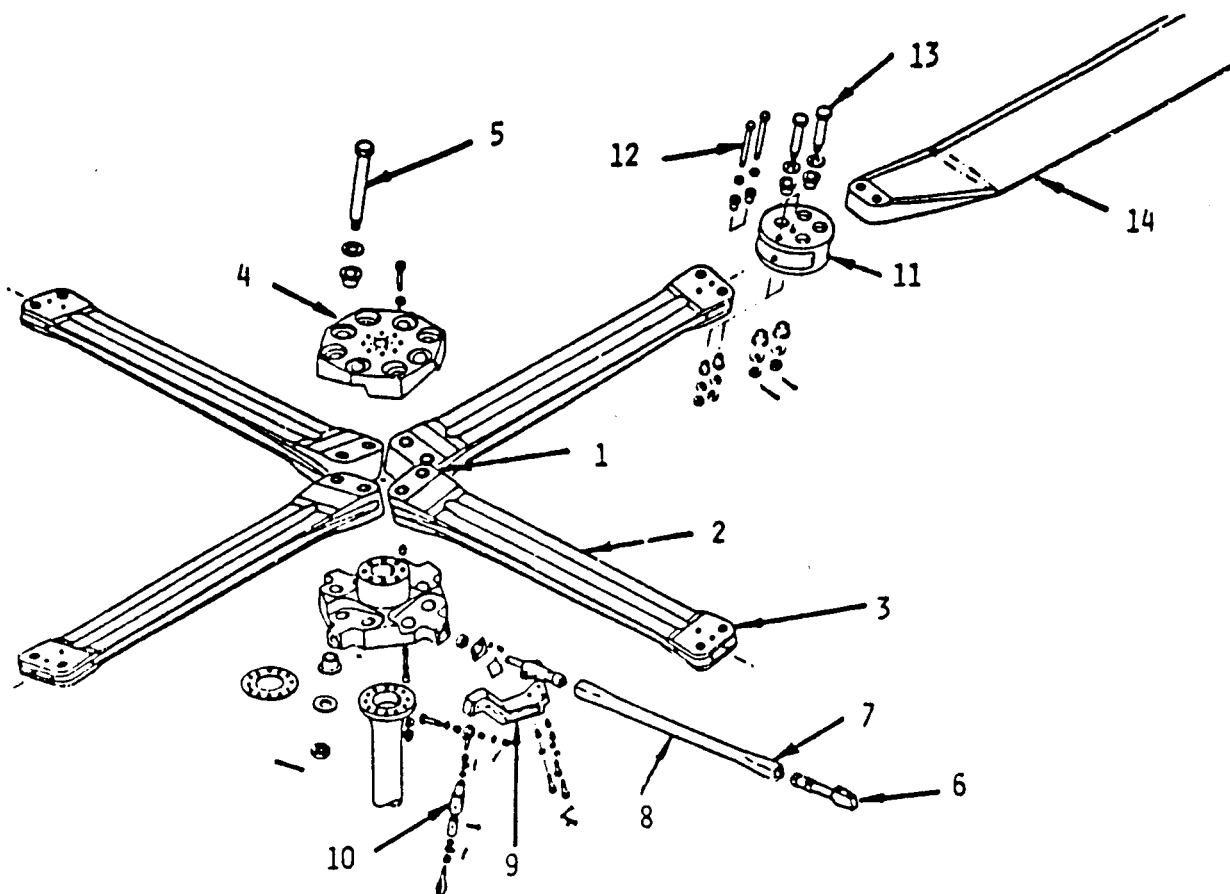
For algebraic simplicity, the coefficients  $B_1$ ,  $B_2$ ,  $e_A$ ,  $k_A^2$  are assumed to be zero in Eqs. (2) to (4). Also, Eq. (1) governing the stretching motion is added to introduce the axial coupling associated with the multiple-load-path blades.

### 3. BEARINGLESS ROTOR BLADES

The bearingless main rotor that was flight tested by Boeing Vertol has three load paths, viz., two fiberglass flexbeams and a filament wound torque tube (Ref. 8). The components of this bearingless main rotor are shown in Fig. 1. The outboard-ends of the flexbeams and torque tube are connected to the blade through a rigid clevis. The inboard-ends of the flexbeams are rigidly connected to the hub. The inboard-end of the torque tube is restrained in torsion by the control system stiffness and is pinned to the hub by a rod-end bearing (translational deflections and bending moments are equal to zero). The idealization of such a blade is shown in Fig. 2. Locations 1, 2, 3 in this figure correspond to the root of the load paths, clevis and tip of the blade respectively. The bearingless main rotor that is currently being designed by Boeing-Vertol has two load paths similar to the one shown in Fig. 3. The formulation presented here is quite general and the computer program is developed for two load paths and can be modified easily for increased number of load paths. For the analysis of multiple-load-path blades, it is important to establish the equilibrium and the compatibility relations across the clevis.

#### 3.1 Equilibrium Across the Clevis

Consider the free-body diagram for the clevis as shown in Fig. 4 and let  $h_{y_i}$  and  $h_{z_i}$  be the y and z coordinates of the ith load path with reference to a coordinate system located at the blade (Point '0').



ITEM	DESCRIPTION
1	FLEXURE, INBOARD ATTACHMENT
2	FLEXURE
3	FLEXURE, OUTBOARD ATTACHMENT
4	STEEL HUB
5	FLEXURE/HUB ATTACHMENT BOLTS
6	TORQUE TUBE, OUTBOARD FITTING
7	TORQUE TUBE, OUTBOARD ATTACHMENT
8	TORQUE TUBE
9	PITCH ARM ASSEMBLY
10	PITCH LINK ASSEMBLY
11	CLEVIS
12	TORQUE TUBE/CLEVIS ATTACHMENT BOLTS
13	BLADE/CLEVIS ATTACHMENT BOLTS
14	ROTOR BLADE ASSEMBLY

Fig. 1. Components of a Bearingless Main Rotor

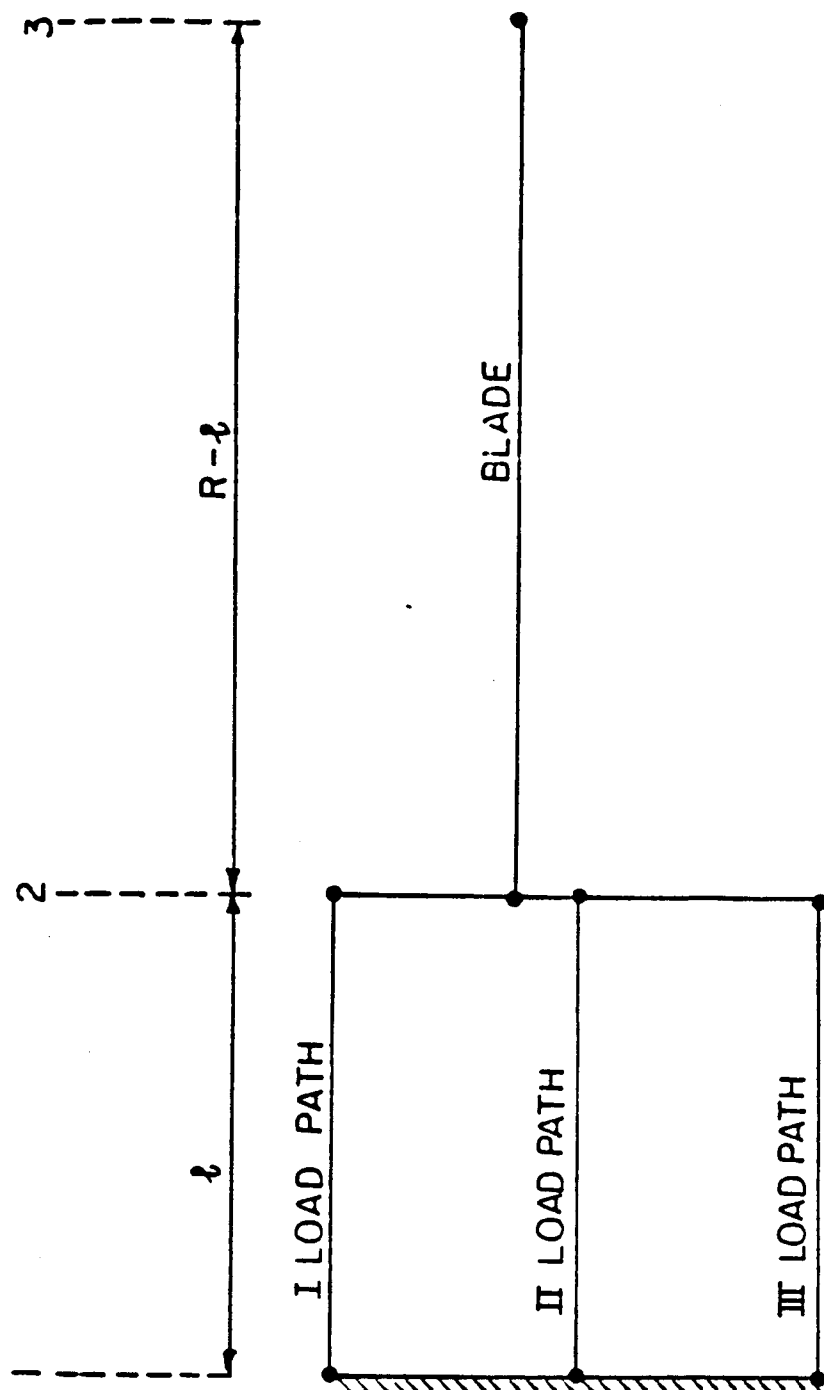


Fig. 2. Model for a Triple-Load-Path Blade

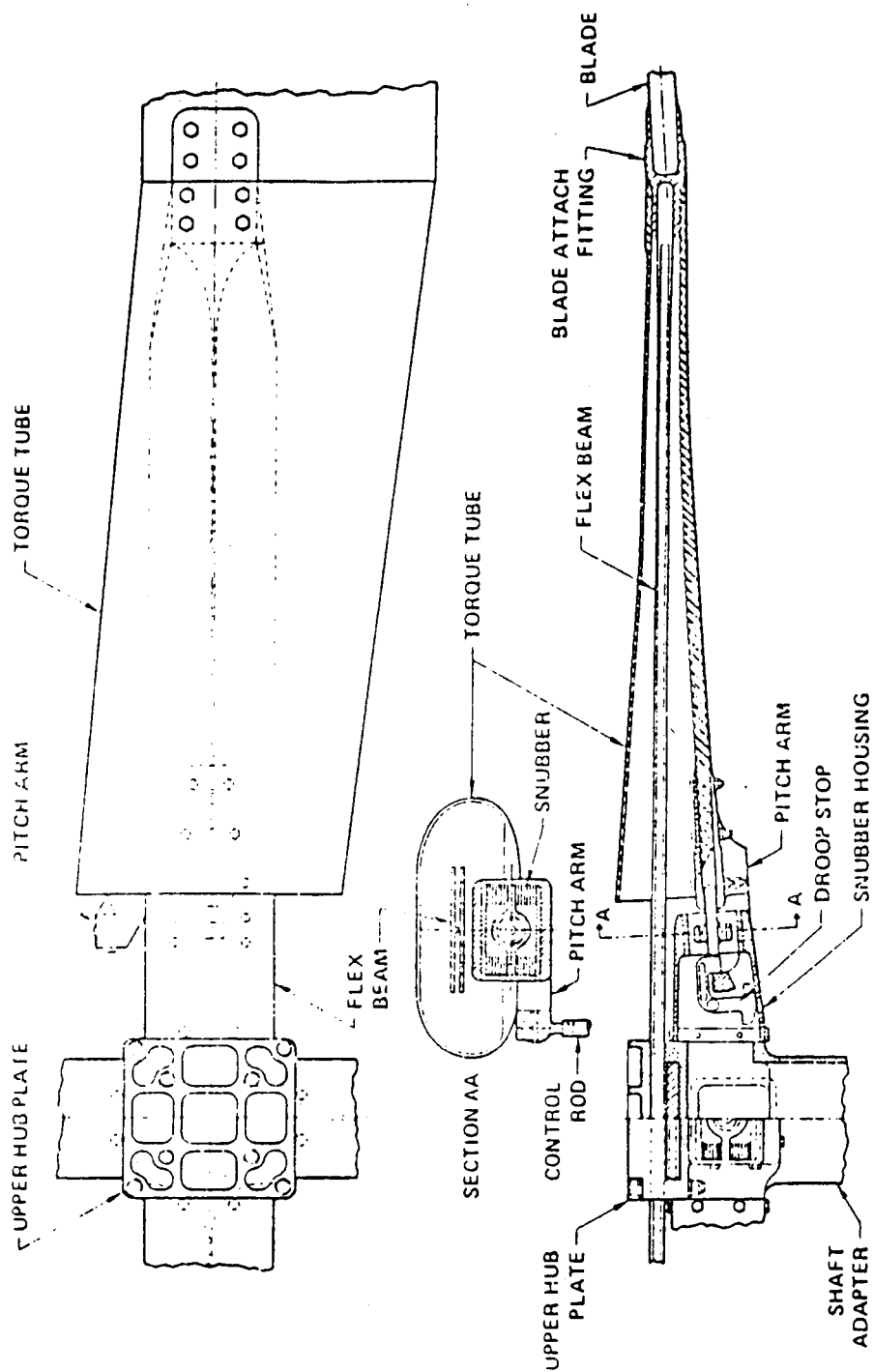


Fig. 3. Bearingless Rotor Concept with Torque Tube Enclosing the Flexibeam

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Force and moment equilibrium require the following relations to be satisfied

$$V_x = \sum_{i=1}^n V_{x_i} \quad (6)$$

$$V_y = \sum_{i=1}^n V_{y_i} \quad (7)$$

$$V_z = \sum_{i=1}^n V_{z_i} \quad (8)$$

$$M_x = \sum_{i=1}^n (M_{x_i} + h_{y_i} V_{z_i} - h_{z_i} V_{y_i}) \quad (9)$$

$$M_y = \sum_{i=1}^n (M_{y_i} + h_{z_i} V_{x_i}) \quad (10)$$

$$M_z = \sum_{i=1}^n M_{z_i} - h_{x_i} V_{x_i} \quad (11)$$

where  $n$  = number of load paths.

### 3.2 Compatibility Across the Clevis

Consider the plane of the clevis as shown in Fig. 5. The axial displacement of the  $i$ th load path can be written as:

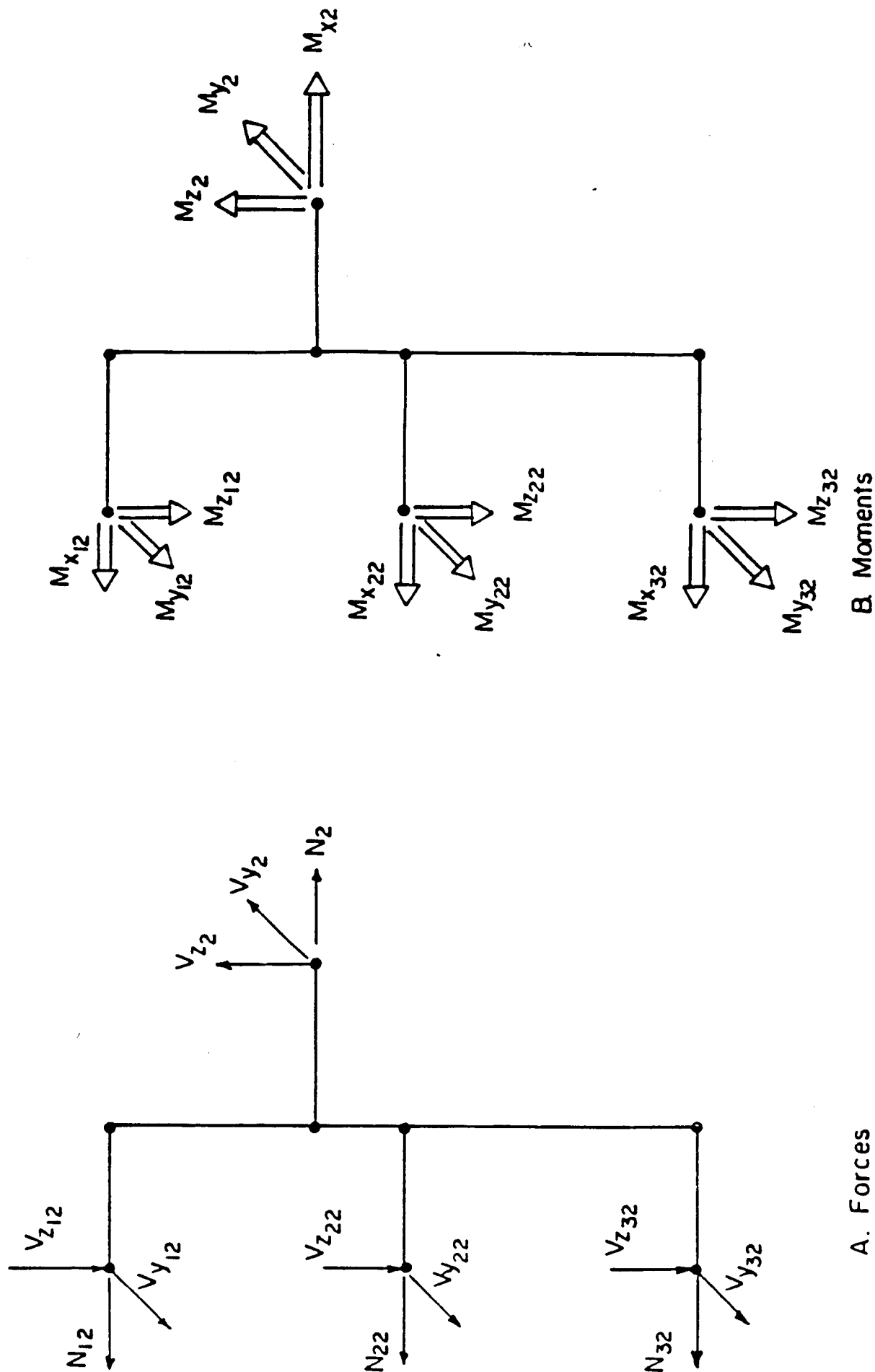


Fig. 4. Free-Body Diagram of the Clevis

$$u_i = u - h_{z_i} w' - h_{y_i} v'$$

Rearrangement of the above equation yields:

$$u = u_i + h_{z_i} w' + h_{y_i} v' \quad (12)$$

The other compatibility conditions consistent with the rigid clevis are

$$w = w_i - h_{y_i} \phi_i \quad (13)$$

$$v = v_i + h_{z_i} \phi_i \quad (14)$$

$$w' = w'_i \quad (15)$$

$$v' = v'_i \quad (16)$$

$$\phi = \phi_i \quad (17)$$

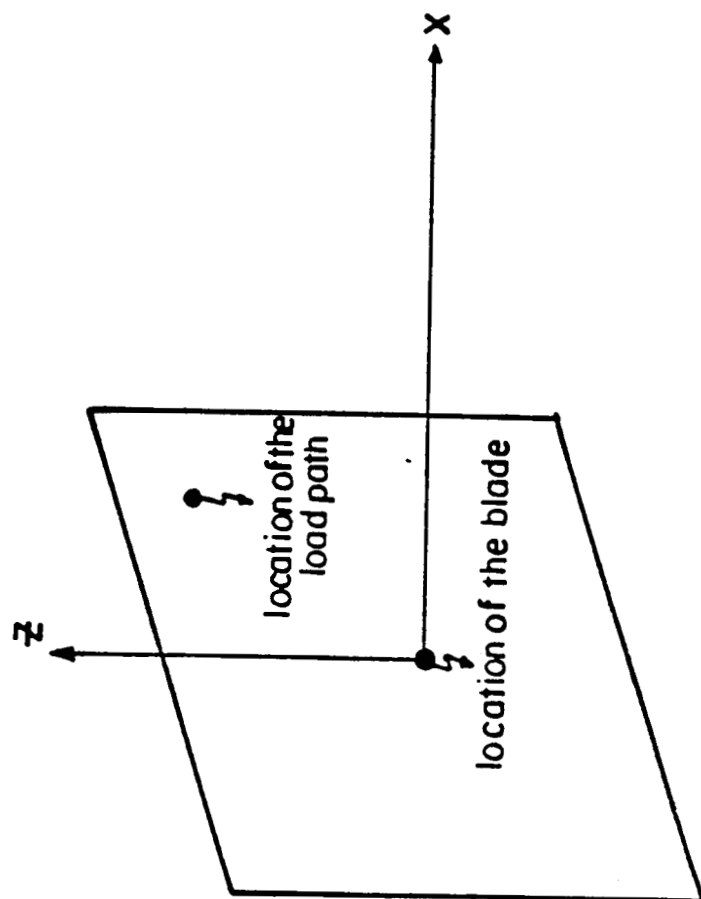


Fig. 5. Plane of the Clevis

#### 4. MODIFIED GALERKIN'S METHOD

The basic equations of motion given by Eqs. (1) to (4) can be written as:

$$\frac{du}{dx} = \frac{V_x}{EA} \quad (18)$$

$$\frac{d\phi}{dx} = \frac{M_x}{GJ} \quad (19)$$

$$\frac{d^2 v}{dx^2} = \frac{-C_{12}}{EI_1 EI_2} M_y + \frac{C_{11}}{EI_1 EI_2} M_z \quad (20)$$

$$\frac{d^2 w}{dx^2} = \frac{C_{22}}{EI_1 EI_2} M_y - \frac{C_{12}}{EI_1 EI_2} M_z \quad (21)$$

$$- \frac{dV_x}{dx} \Omega^2 \mu = \omega^2 \mu \quad (22)$$

$$\begin{aligned} & - \frac{dM_x}{dx} + \Omega^2 m (k_{m2}^2 - k_{m1}^2) \cos 2\beta\phi \\ & - \Omega^2 m x (-\sin\beta v' + \cos\beta w') + \Omega^2 m \sin\beta v \\ & = \omega^2 m k_m^2 \phi - \omega^2 m \sin\beta v + \omega^2 m \cos\beta w \end{aligned} \quad (23)$$

$$\begin{aligned} & \frac{d^2 M_y}{dx^2} - (T \frac{dw}{dx})' - (\Omega^2 m x \cos\beta\phi)' \\ & = \omega^2 m w + \Omega^2 m \cos\beta\phi \end{aligned} \quad (24)$$

$$\begin{aligned}
& \frac{d^2 M_z}{dx^2} - \left( T \frac{dv}{dx} \right)' + (\Omega^2 m \cos \beta \phi)' \\
& + \Omega^2 m \sin \beta \phi - \Omega^2 m v = \omega^2 m v - \omega^2 m \sin \beta \phi
\end{aligned} \tag{25}$$

where

$$C_{11} = EI_1 \cos^2 \beta + EI_2 \sin^2 \beta$$

$$C_{12} = (EI_2 - EI_1) \cos \beta \sin \beta$$

$$C_{22} = EI_1 \sin^2 \beta + EI_2 \cos^2 \beta$$

A convenient set of functions for Galerkin's method are selected with the following properties

1. Satisfy the root boundary conditions of the load paths
2. Satisfy the clevis compatibility conditions
3. Satisfy the blade tip conditions

The deflections and forces are expanded as finite series in terms of the selected functions as shown below. The choice of functions and their modifications are discussed in Chapter 5.

$$u = \sum_{n=1}^N a_{1n} u_n$$

$$u_i = \sum_{n=1}^N a_{1n} u_{n_i}$$

$$v = \sum_{n=1}^N a_{2n} v_n$$

$$v_i = \sum_{n=1}^N a_{2n} v_{n_i}$$

$$w = \sum_{n=1}^N a_{3n} w_n$$

$$w_i = \sum_{n=1}^N a_{3n} w_{n_i}$$

$$\phi = \sum_{n=1}^N a_{4n} \phi_n$$

$$\phi_i = \sum_{n=1}^N a_{4n} \phi_{n_i}$$

(26)

$$v_x = \sum_{n=1}^N b_{1n} v_{xn}$$

$$v_{x_i} = \sum_{n=1}^N b_{1n} v_{xn_i}$$

$$M_z = \sum_{n=1}^N b_{2n} M_{zn}$$

$$\begin{aligned}
 M_{z_i} &= \sum_{n=1}^N b_{2n} M_{zn_i} \\
 M_y &= \sum_{n=1}^N b_{3n} M_{yn_i} \\
 M_{y_i} &= \sum_{n=1}^N b_{3n} M_{yn_i} \\
 M_x &= \sum_{n=1}^N b_{4n} M_{xn_i} \\
 M_{x_i} &= \sum_{n=1}^N b_{4n} M_{xn_i}
 \end{aligned}
 \tag{27}$$

The satisfaction of the following equations is the modified Galerkin's method for multiple-load-path blades which is equivalent to the quotient method described in Ref. 6. (N = Number of functions selected; n = number of load paths)

$$\sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} [b_{3j} M''_{yj_i} - a_{3j} (T_i w'_{j_i})' + a_{4j} (\Omega^2 m_i e_i x \cos \beta_i \phi_{j_i})'] w_{k_i} dx \right\}$$



$$\begin{aligned}
& + \sum_{j=1}^N \left\{ \int_{\ell}^R [b_{3j} M''_{y_j} - a_{3j} (T w'_j)' \right. \\
& \quad \left. + a_{4j} (\Omega^2 m_{ex} \cos \beta \phi_j)' ] w_k dx \right\} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (-b_{3j} M'_{y_{ji}} + a_{3j} T_i w'_{ji} + a_{4j} \Omega^2 m_i e_i x \cos \beta_i \phi_{ji}) \right. \right. \\
& \quad \left. \left. - (-b_{3j} M'_{y_j} + a_{3j} T w_j + a_{4j} \Omega^2 m_{ex} \cos \beta \phi_j) \right\} w_k \right]_{x=\ell} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (b_{3j} M_{y_{ji}} - h_{z_i} b_{1j} V_{x_{ji}}) - b_{3j} M_{y_j} \right\} w'_k \right]_{x=\ell} \\
& = \omega^2 \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} m_i (a_{3j} w_{ji} + a_{4j} e_i \cos \beta_i \phi_{ji}) w_{k_i} dx \right\} \right] \\
& + \omega^2 \left[ \sum_{j=1}^N \int_{\ell}^R m (a_{3j} w_j + a_{4j} e \cos \beta \phi_j) w_k dx \right]
\end{aligned} \tag{28}$$

where

$$k = 1, 2, 3, \dots, N$$

$$\begin{aligned}
& \sum_{j=1}^N < \sum_{i=1}^n \left\{ \int_0^{\ell} [b_{2j} M''_{zj_i} - a_{2j} (T_i v'_{j_i})' + a_{4j} (\Omega^2 m_i e_i x \sin \beta_i \phi_{j_i})' \right. \\
& \quad \left. + a_{4j} \Omega^2 m_i e_i \sin \beta_i \phi_{j_i} - a_{2j} \Omega^2 m_i v_{j_i}] v_{k_i} dx \right\} > \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R [b_{2j} M''_{zj} - a_{2j} (T v'_j)' + a_{4j} (\Omega^2 m e x \sin \beta \phi_j)' \right. \\
& \quad \left. + a_{4j} \Omega^2 m e \sin \beta \phi_j - a_{2j} \Omega^2 m v_j] v_k dx \right\} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (-b_{2j} M'_{zj_i} + a_{2j} T_j v'_{j_i} - a_{4j} \Omega^2 m_i e_i x \sin \beta_i \phi_{j_i}) \right. \right. \\
& \quad \left. \left. - (-b_{2j} M'_{zj} + a_{2j} T v'_j - a_{4j} \Omega^2 m e x \sin \beta \phi_j) \right\} v_k \right]_{x=\ell} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (b_{2j} M_{zj_i} - h_{x_i} b_{1j} v_{xj_i}) - b_{2j} M_{zj} \right\} v'_k \right]_{x=\ell} \\
& = \omega^2 \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} m_i (a_{2j} v_{j_i} - a_{4j} e_i \sin \beta_i \phi_{j_i}) v_{k_i} dx \right\} \right] \\
& + \omega^2 \left[ \sum_{j=1}^N \int_{\ell}^R m (a_{2j} v_j - a_{4j} e \sin \beta \phi_j) v_k dx \right] \tag{29}
\end{aligned}$$

where  $k = 1, 2, 3, \dots, N$

$$\begin{aligned}
& \sum_{j=1}^N \left[ \sum_{i=1}^n \int_0^{\ell} \left\{ -b_{4j} M'_{xj_i} + \Omega^2 m_i e_i x (-a_{2j} \cos \beta_i v'_{j_i} + a_{3j} \sin \beta_i w'_{j_i}) \right. \right. \\
& \quad \left. \left. + a_{2j} \Omega^2 m_i e_i \sin \beta_i v_{j_i} + a_{4j} \Omega^2 m_i (k_{m_2}^2 - k_{m_1}^2) \cos 2\beta_i \phi_{j_i} \right\} \phi_{k_i} dx \right] \\
& + \sum_{j=1}^N \left[ \int_{\ell}^R \left\{ -b_{4j} M'_{xj} + \Omega^2 m e x (-a_{2j} \cos \beta v'_j + a_{3j} \sin \beta w'_j) \right. \right. \\
& \quad \left. \left. + a_{2j} \Omega^2 m e \sin \beta v_j + a_{4j} \Omega^2 m (k_{m_2}^2 - k_{m_1}^2) \cos 2\beta \phi_j \right\} \phi_k dx \right] \\
& + < \sum_{j=1}^N \left\{ \sum_{i=1}^n [b_{4j} M'_{xj_i} + h_{y_i} (-b_{3j} M'_{yj_i} + a_{3j} T_i w'_{j_i} \right. \\
& \quad + a_{4j} \Omega^2 m_i e_i x \cos \beta_i \phi_{j_i}) - h_{z_i} (-b_{2j} M'_{zj_i} + a_{2j} T_i v'_{j_i} \\
& \quad \left. - a_{4j} \Omega^2 m_i e_i x \sin \beta_i \phi_{j_i})] - b_{4j} M'_{xj} \right\} \phi_k >_{x=\ell} \\
& = \omega^2 \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} m_i (a_{4j} k_{m_i}^2 \phi_{j_i} + a_{3j} e_i \cos \beta_i w_{j_i} \right. \right. \\
& \quad \left. \left. - a_{2j} e_i \sin \beta_i v_{j_i}) \phi_{k_i} dx \right\} \right] \\
& + \omega^2 \left\{ \sum_{j=1}^N \int_{\ell}^R m (a_{4j} k_m^2 \phi_j + a_{3j} e \cos \beta w_j \right. \\
& \quad \left. - a_{2j} e \sin \beta v_j) \phi_k dx \right\}
\end{aligned} \tag{30}$$

where  $k = 1, 2, 3, \dots, N$

$$\begin{aligned}
 & \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (-b_{1j} v_{xj_i} - a_{1j} \Omega^2 m_i u_{j_i}) u_{k_i} dx \right\} \\
 & + \sum_{j=1}^N \left\{ \int_{\ell}^R (-b_{1j} v'_{xj} - a_{1j} \Omega^2 m u_j) u_k dx \right\} \\
 & + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (b_{1j} v_{xj_i}) - b_{1j} v_{xj} \right\} u_k \right]_{x=\ell} \\
 & = \omega^2 \left[ \sum_{j=1}^N \left\{ \int_0^{\ell} m_i a_{1j} u_{j_i} u_{k_i} dx \right\} \right] \\
 & + \omega^2 \left[ \int_{\ell}^R m a_{1j} u_j u_k dx \right]
 \end{aligned} \tag{31}$$

where  $k = 1, 2, 3, \dots, N$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} \left( a_{1j} u_{ji} - \frac{b_{1j} v_{xji}}{AE_i} \right) v_{xk_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R \left( a_{1j} u_j - \frac{b_{1j} v_{xj}}{EA} \right) v_{xk} dx \right\} = 0 \quad (32)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} \left( a_{4j} \phi_{ji} - \frac{b_{4j} M_{xji}}{GJ_i} \right) M_{xk_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R \left( a_{4j} \phi_j - \frac{b_{4j} M_{xj}}{GJ} \right) M_{xk} dx \right\} = 0 \quad (33)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} \left( a_{2j} v''_{ji} + \frac{C_{12}}{EI_1 EI_2} b_{3j} M_{yji} \right. \right. \\
& \quad \left. \left. - \frac{C_{11}}{EI_1 EI_2} b_{2j} M_{zji} \right) M_{zk_i} dx \right\} + \\
& \sum_{j=1}^N \left\{ \int_{\ell}^R \left( a_{2j} v''_j + \frac{C_{12}}{EI_1 EI_2} b_{3j} M_{yj} \right) \right. \\
& \quad \left. - \frac{C_{11}}{EI_1 EI_2} b_{2j} M_{zj} \right) M_{zk} dx \right\} = 0 \quad (34)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^l (a_{3j} w_{ji}'' - \frac{c_{22}}{EI_1 EI_2} b_{3j} M_{yji} \right. \\
& \quad \left. + \frac{c_{12}}{EI_1 EI_2} b_{2j} M_{zji} ) M_{yk_i} dx \right\} + \\
& \sum_{j=1}^N \left\{ \int_l^R (a_{3j} w_j'' - \frac{c_{22}}{EI_1 EI_2} b_{3j} M_{yj} \right. \\
& \quad \left. + \frac{c_{12}}{EI_1 EI_2} b_{2j} M_{zj} ) M_{yk} dx \right\} = 0
\end{aligned} \tag{35}$$

## 5. SELECTION OF FUNCTIONS

### 5.1 Basic Functions

The selection of functions for multiple-load-path blades with fixed-conditions for load path members (inboard-ends) is presented in this section. The other inboard-end conditions can be incorporated easily in a straightforward manner. The CAMRAD program uses nonrotating uniform modes for the Galerkin's method; therefore, the same modes are employed for the multiple-load-path blades for uniformity. The non-rotating modes for uniform cantilever beams are given by:

1. Bending (Both Flapwise and Chordwise)

a) Mode shapes

$$\alpha_r(x) = e_r \left( \sin \alpha_r \frac{x}{R} - \sinh \alpha_r \frac{x}{R} \right) + g_r \left( \cos \alpha_r \frac{x}{R} - \cosh \alpha_r \frac{x}{R} \right) \quad (36)$$

where

$$e_r = -(\sin \alpha_r - \sinh \alpha_r) / 2 \sin \alpha_r \sinh \alpha_r$$

$$g_r = (\cos \alpha_r + \cosh \alpha_r) / 2 \sin \alpha_r \sinh \alpha_r$$

$r$  = rth mode shape

$$\alpha_1 = 1.875, \alpha_2 = 4.694, \alpha_3 = 7.855, \alpha_4 = 10.996$$

$$\alpha_5 = 14.137, \alpha_6 = 17.279, \alpha_7 = 20.420, \alpha_8 = 23.562$$

$$\alpha_9 = 26.704, \alpha_{10} = 29.845$$

## b) Frequencies

$$\omega_r = \alpha_r^2 \sqrt{EI/mR^4}$$

## 2) Stretching and Twisting

## a) Mode shapes

$$\alpha_r(x) = \sin(\delta_r \frac{x}{R}) \quad (37)$$

$$\text{where } \delta_r = (2r-1) \frac{\pi}{2}$$

## b) Frequencies

$$\omega_r = \delta_r \sqrt{EA/mR^2}, \text{ Stretching}$$

$$\omega_r = \delta_r \sqrt{GJ/mR^2}, \text{ Twisting}$$

The idealization for a two-load-path blade is shown in Fig. 6.

The blade is divided into three regions

Region I:  $0 \leq x \leq \ell_1$ , all load path members

Region II:  $\ell \leq x \leq R$ , blade

Region III:  $\ell_1 \leq x \leq \ell$ , load path members

For regions I and II, the basic mode functions given by Eqs. (36) and (37) are used. It is therefore necessary to develop the clevis admissible comparison functions. At this stage the following two options are available.



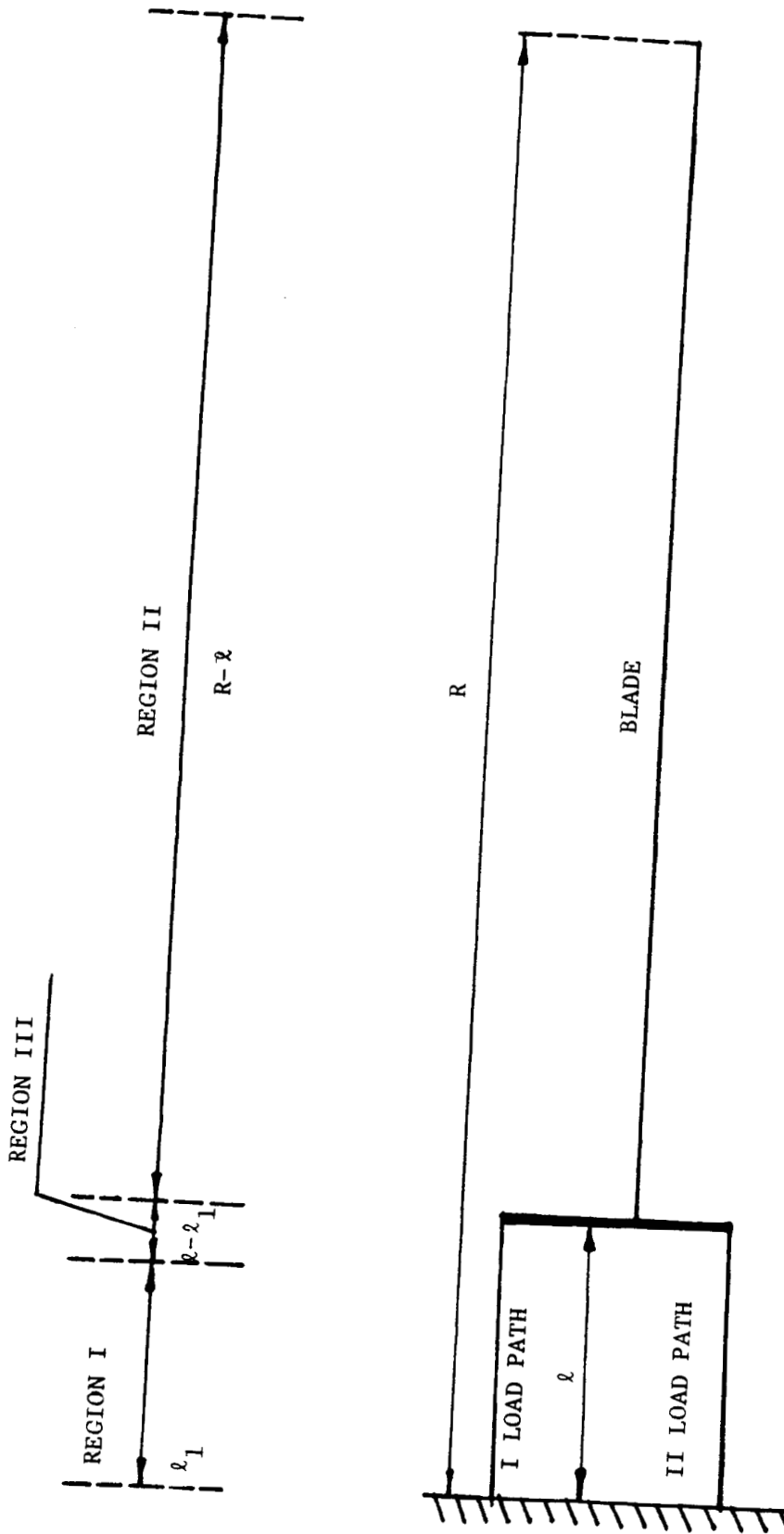


Fig. 6. Division of the Regions

1. Satisfy geometric compatibility only and account for the force conditions in the Galerkins' procedure.
2. Satisfy both geometric and force conditions.

Option 1 is selected here because the clevis compatibility functions do not depend on the eigenvalues while the clevis admissible comparison functions depend on the eigenvalues.

## 5.2 Clevis Compatibility Functions

### 1. Flapwise Bending

Assume the functions to be of the following form:

$$w_{r_i} = a_{0r_i}^1 + a_{1r_i}^1 x + a_{2r_i}^1 x^2 + a_{3r_i}^1 x^3 \quad (38)$$

The coefficients in the equation are determined by satisfying the displacement and slope continuity at  $x = \ell_1$  (see Fig. 6) and geometric compatibility at the clevis [Eqs. (13) and (14)].

The resulting equations are given by:

$$a_{0r_i}^1 + a_{1r_i}^1 \ell_1 + a_{2r_i}^1 \ell_1^2 + a_{3r_i}^1 \ell_1^3 = w_{r_i}(\ell_1)$$

$$a_{1r_i}^1 + 2a_{2r_i}^1 \ell_1 + 3a_{3r_i}^1 \ell_1^2 = w'_{r_i}(\ell_1)$$

$$a_{0r_i}^1 + a_{1r_i}^1 \ell + a_{2r_i}^1 \ell^2 + a_{3r_i}^1 \ell^3 = w_r(\ell) + h_{y_i} \phi_r(\ell)$$

$$a_{1r_i}^1 + 2a_{2r_i}^1 \ell + 3a_{3r_i}^1 \ell^2 = w'_r(\ell)$$

Arranging the above equations into a matrix form yields:

Arranging the above equations into a matrix form yields:

$$\begin{Bmatrix} a_{0r_i}^1 \\ a_{1r_i}^1 \\ a_{2r_i}^1 \\ a_{3r_i}^1 \end{Bmatrix} = \begin{bmatrix} 1 & \ell_1 & \ell_1^2 & \ell_1^3 \\ 0 & 1 & 2\ell_1 & 3\ell_1^2 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^2 \end{bmatrix}^{-1} \begin{Bmatrix} w_{r_i}(\ell_1) \\ w'_{r_i}(\ell_1) \\ w_r(\ell) + h_{y_i} \phi_r(\ell) \\ w'_r(\ell) \end{Bmatrix} \quad (39)$$

where

subscript  $r$  = mode index

subscript  $i$  = load path number index

superscript 1 = flapwise bending

## 2. Chordwise Bending

Assume the functions to be of the following form:

$$v_{r_i} = a_{0r_i}^2 + a_{1r_i}^2 x + a_{2r_i}^2 x^2 + a_{3r_i}^2 x^3 \quad (40)$$

By employing a similar procedure used for the flapwise bending the following equation can be obtained for the chordwise bending clevis compatibility function.

$$\begin{Bmatrix} a_{0r_i}^2 \\ a_{1r_i}^2 \\ a_{2r_i}^2 \\ a_{3r_i}^2 \end{Bmatrix} = \begin{bmatrix} 1 & \ell_1 & \ell_1^2 & \ell_1^3 \\ 0 & 1 & 2\ell_1 & 3\ell_1^2 \\ 1 & \ell & \ell^2 & \ell^3 \\ 0 & 1 & 2\ell & 3\ell^2 \end{bmatrix}^{-1} \begin{Bmatrix} v_{r_i}(\ell_1) \\ v'_{r_i}(\ell_1) \\ v_r(\ell) - h_{z_i} \phi_r(\ell) \\ v'_r(\ell) \end{Bmatrix} \quad (41)$$

where superscript 2 indicates the chordwise bending.

## 3. Twisting

Assume the function to be of the following form:

$$\phi_{r_i} = b_{0r_i}^1 + b_{1r_i}^1 x \quad (42)$$

By satisfying the continuity at  $x = \ell_1$  and compatibility at the clevis the following relation can be obtained for the coefficients in Eq. (42).

$$\begin{Bmatrix} b_{0r_i}^1 \\ b_{1r_i}^1 \end{Bmatrix} = \begin{bmatrix} 1 & \ell_1 \\ 1 & \ell \end{bmatrix}^{-1} \begin{Bmatrix} \phi_{r_i}(\phi_1) \\ \phi_r(\ell) \end{Bmatrix} \quad (43)$$

The superscript 1 indicates twisting.

#### 4. Stretching

Assume the functions to be of the following form:

$$u_{r_i} = b_{0r_i}^2 + b_{1r_i}^2 x \quad (44)$$

The coefficients in the above equations are computed by

$$\begin{Bmatrix} b_{0r_i}^2 \\ b_{1r_i}^2 \end{Bmatrix} = \begin{bmatrix} 1 & \ell_1 \\ 1 & \ell \end{bmatrix}^{-1} \begin{Bmatrix} u_{r_i}(\ell_1) \\ u(\ell) - h_{z_i} w'(\ell) - h_{y_i} v'(\ell) \end{Bmatrix} \quad (45)$$

The superscript 2 in the above equation indicates stretching.

The treatment for the selection of function is completed so far for the deflections, and it is necessary to establish the criteria for the forces. For simplicity the functions used for the forces are as follows.

$$\left. \begin{aligned}
 M_x &= \phi' \\
 M_y &= w'' \\
 M_z &= v'' \\
 V_x &= u'
 \end{aligned} \right\} \quad (46)$$

For all regions (I, II and III), the basic functions defined in Eqs. (36) and (37) are employed while calculating the force functions given by Eq. (46). If the following equations are satisfied for the force function, then the modified Galerkin's procedure used in CAMRAD would reduce to the standard Galerkin form.

$$\left. \begin{aligned}
 M_x &= GJ\phi' \\
 M_y &= (EI_1 \cos^2 \beta + EI_2 \sin^2 \beta)w'' + (EI_2 - EI_1) \cos \beta \sin \beta v'' \\
 M_z &= (EI_1 \sin^2 \beta + EI_2 \cos^2 \beta)v'' + (EI_2 - EI_1) \cos \beta \sin \beta w'' \\
 V_x &= EAu'
 \end{aligned} \right\} \quad (47)$$

In this situation, it is desirable to use fifth degree polynomials for bending degrees-of-freedom and quadratic functions for the twisting and stretching degrees-of-freedom for generating the clevis compatibility functions. However, the standard Galerkin form is not employed for parallel development with the CAMRAD approaches.

## 6. TENSION CALCULATIONS

The rotating blade mode shapes calculations are based on the linear formulation, and therefore the tension coefficient  $T$  appearing in Eqs. (2) and (3) should be known in advance; otherwise these equations will be nonlinear. Equation (5) can be employed for calculation of tension in the blade. The tensions in the load paths are calculated by assuming that the load paths are coincident with the blade at the clevis, i.e.,  $h_{y_i} = h_{z_i} = 0$ . The tensions corresponding to this case are given by (Ref. 9):

$$T_i(x) = \int_x^{\ell} \Omega^2 m_i x dx + T_i(\ell) \quad (48)$$

where

$$T_i(\ell) = \frac{\sum_{j=1}^n a_j}{\sum_{k=1}^n \left( \prod_{j=1}^{n,k} a_j \right)} \quad (49)$$

$$\prod_{j=1}^{n,i} a_j = a_1 a_2 \dots a_n / a_i \quad (50)$$

$$a_i = \int_0^{\ell} \frac{dx}{EA_i} \quad (51)$$

It is to be noted that the tensions given by Eq. (48) correspond to the coincident load path case ( $h_{y_i} = h_{z_i} = 0$ ), and for the noncoincident case ( $h_{y_i}$  and/or  $h_{z_i} \neq 0$ ) the modified Galerkin's method is applied starting with the coincident load path case.

#### Modified Galerkin's Method for Tensions

The free-body diagram for tension calculations in the load path with blade detached is shown in Fig. 7. The equilibrium conditions forces at the clevis are given by:

$$\sum_{i=1}^N V_{x_i} = T(\ell) = \int_{\ell}^R \Omega^2 m x dx$$

$$\sum_{i=1}^n V_{y_i} = 0$$

$$\sum_{i=1}^n (M_{x_i} + h_{y_i} V_{z_i} - h_{z_i} V_{y_i}) = 0$$

$$\sum_{i=1}^n (M_{y_i} + h_{z_i} V_{x_i}) = 0$$

$$\sum_{i=1}^n (M_{z_i} - h_{y_i} V_{x_i}) = 0$$



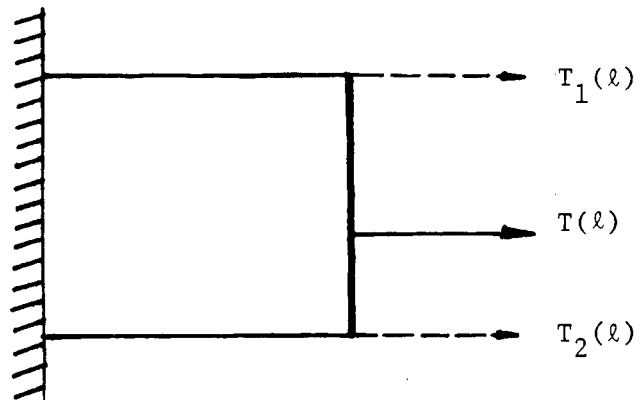


Fig. 7. Tension Distribution

A convenient set of functions are selected for the load path members with the following properties.

1. Satisfy the root boundary conditions of the load paths.
2. Satisfy the clevis compatibility conditions.

The deflections and forces in the load path members are expanded as finite series in terms of the selected functions as shown below

$$u_i = \sum_{n=1}^N a_{1n} u_{n_i}$$

$$v_i = \sum_{n=1}^N a_{2n} v_{n_i}$$

$$w_i = \sum_{n=1}^N a_{3n} w_{n_i}$$

$$\phi_i = \sum_{n=1}^N a_{4n} \phi_{n_i}$$

$$V_{x_i} = \sum_{n=1}^N b_{1n} V_{xn_i}$$

$$M_{z_i} = \sum_{n=1}^N b_{2n} M_{zn_i}$$

(52)

$$M_{y_i} = \sum_{n=1}^N b_{3n} M_{yn_i}$$

$$M_{x_i} = \sum_{n=1}^N b_{4n} M_{xn_i}$$

The adaptation of Galerkin's method yields the following equations:

$$\begin{aligned} & \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^l [b_{3j} M''_{yj_i} - a_{3j} (T_i w'_{j_i})' \right. \\ & \quad \left. + a_{4j} (\Omega^2 m_i e_i x \cos \beta_i \phi_{j_i})' ] w_{k_i} dx \right\} \\ & + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (-b_{3j} M'_{yj_i} + a_{3j} T_i w'_{j_i} \right. \right. \\ & \quad \left. \left. + a_{4j} \Omega^2 m_i e_i x \cos \beta_i \phi_{j_i}) w_{k_i} \right\} \right]_{x=l} \\ & + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (b_{3j} M_{yj_i} - h_{z_i} b_{1j} v_{xj_i}) w'_{k_i} \right\} \right]_{x=l} = 0 \end{aligned} \quad (53)$$

$$\begin{aligned}
& \sum_{n=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} [b_{2j} M''_{zj_i} - a_{2j} (T_i v'_{j_i})' + a_{4j} (\Omega^2_{m_i} e_i x \sin \beta_i \phi_{j_i})' \right. \\
& \quad \left. + a_{4j} \Omega^2_{m_i} e_i \sin \beta_i \phi_{j_i} - a_{2j} \Omega^2_{m_i} v_{j_i}] v_{k_i} dx \right\} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (-b_{2j} M'_{zj} + a_{2j} T_i v'_{j_i} \right. \right. \\
& \quad \left. \left. - a_{4j} \Omega^2_{m_i} e_i x \sin \beta_i \phi_{j_i}) v_k \right\} \right]_{x=\ell} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (b_{2j} M'_{zj_i} - h_{y_i} b_{1j} v_{xj_i}) v'_k \right\} \right]_{x=\ell} = 0 \quad (54)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left[ \sum_{i=1}^n \int_0^{\ell} \left\{ -b_{4j} M'_{xj_i} + \Omega^2_{m_i} e_i x (-a_{2j} \cos \beta_i v'_{j_i} \right. \right. \\
& \quad \left. \left. + a_{3j} \sin \beta_i w'_{j_i}) + a_{2j} \Omega^2_{m_i} e_i \sin \beta_i v_{j_i} \right. \right. \\
& \quad \left. \left. + a_{4j} \Omega^2_{m_i} (k_{m_2}^2 - k_{m_1}^2) \cos 2\beta_i \phi_{j_i} \right\} \phi_{k_i} dx \right]
\end{aligned}$$

$$\begin{aligned}
& + < \sum_{j=1}^N \left\{ \sum_{i=1}^n [b_{4j} M_{xj_i} + h_{y_i} (-b_{3j} M'_{yj_i} + a_{3j} T_i w'_{j_i} \right. \\
& \quad + a_{4j} \Omega^2 m_i e_i x \cos \beta_i \phi_{j_i}) - h_{z_i} (-b_{2j} M'_{zj_i} \\
& \quad \left. + a_{2j} T_i v'_{j_i} - a_{4j} \Omega^2 m_i e_i x \sin \beta_i \phi_{j_i})] \phi_k \right\}_{x=\ell} = 0 \quad (55)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^\ell (-b_{1j} v'_{xj_i} - a_{1j} \Omega^2 m_i u_{j_i}) u_{k_i} dx \right\} \\
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n b_{1j} v_{xj_i} u_k \right\} \right]_{x=\ell} = T(\ell) u_k(\ell) \quad (56)
\end{aligned}$$

$$\sum_{n=1}^N \left\{ \sum_{i=1}^n \int_0^\ell (a_{1j} u_{j_i} - \frac{b_{1j} v_{xj_i}}{EA_i}) v_{xk_i} dx \right\} = 0 \quad (57)$$

$$\sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^\ell (a_{4j} \phi_{j_i} - \frac{b_{4j} M_{xj_i}}{GJ_i}) M_{xk_i} dx \right\} = 0 \quad (58)$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^\ell (a_{2j} v''_{j_i} + \frac{c_{12}}{EI_1 EI_2} b_{3j} M_{yj_i} \right. \\
& \quad \left. - \frac{c_{11}}{EI_1 EI_2} b_{2j} M_{zj_i}) M_{zk_i} dx \right\} = 0 \quad (59)
\end{aligned}$$

$$\sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} \left( a_{3j} w''_{ji} - \frac{c_{22}}{EI_1 EI_2} b_{3j} M_{yj_i} + \frac{c_{12}}{EI_1 EI_2} b_{2j} M_{zj_i} \right) M_{yk_i} dx \right\} = 0 \quad (60)$$

where  $c_{11}$ ,  $c_{12}$ ,  $c_{22}$  are defined in Eq. (35). The solution of Eqs. (53) to (60) yields  $T_i(\ell)$  [which is  $V_{x_i}(\ell)$ ], and from Eq. (48) the tension coefficients in the load path members can be computed. At the most, two or three iterations may be needed starting with the coincident load path tensions to solve the nonlinear problem. The functions developed in Chapter 5 are used to solve the present problem also.

## 7. SOFTWARE DEVELOPMENT

### 7.1 Equations of Motion

The basic equations of motion given by Eqs. (18) to (25) are nondimensionalized as shown below.

$$\frac{d\bar{u}}{d\bar{x}} = \frac{\bar{V}_x}{\bar{EA}} \quad (61)$$

where  $\bar{u} = u/R$ ;  $\bar{x} = x/R$ ;  $\bar{EA} = EAR^2/EI_R$   
 $\bar{V}_x = V_x R^2/EI_R$

$$\frac{d\bar{\phi}}{d\bar{x}} = \frac{\bar{M}_x}{\bar{GJ}} \quad (62)$$

where  $\bar{GJ} = GJ/EI_R$ ;  $\bar{M}_x = M_x R/EI_R$

$$\frac{d^2 \bar{v}}{d\bar{x}^2} = \frac{-\bar{C}_{12}}{EI_1 EI_2} \bar{M}_y + \frac{\bar{C}_{11}}{EI_1 EI_2} \bar{M}_z \quad (63)$$

where  $\bar{C}_{12} = (\bar{EI}_2 - \bar{EI}_1) \cos \bar{\beta} \sin \bar{\beta}$

$$\bar{C}_{11} = \bar{EI}_1 \cos^2 \bar{\beta} + \bar{EI}_2 \sin^2 \bar{\beta}$$

$$\bar{EI}_1 = EI_1/EI_R; \bar{EI}_2 = EI_2/EI_R; \bar{v} = v/R$$

$$\bar{\beta} = \beta; \bar{M}_y = M_y R/EI_R; \bar{M}_z = M_z R/EI_R$$

$$\frac{d^2 \bar{w}}{d\bar{x}^2} = \frac{\bar{c}_{22}}{\bar{EI}_1 \bar{EI}_2} \bar{M}_y - \frac{\bar{c}_{12}}{\bar{EI}_1 \bar{EI}_2} \bar{M}_z \quad (64)$$

where  $\bar{c}_{22} = \bar{EI}_1 \sin^2 \bar{\beta} + \bar{EI}_2 \cos^2 \bar{\beta}$   
 $\bar{w} = w/R$

$$\frac{-d\bar{v}}{d\bar{x}} - \bar{\Omega}^2 \bar{m} \bar{u} = \omega^2 \bar{m} \bar{u} \quad (65)$$

where  $\bar{\Omega}^2 = \Omega^2 m_R R^4 / EI_R$ ;  $\bar{m} = m/m_R$   
 $\bar{\omega}^2 = \omega^2 m_R R^4 / EI_R$

$$\begin{aligned} -\frac{d\bar{M}_x}{d\bar{x}} + \bar{\Omega}^2 m (\bar{k}_{m_2}^2 - \bar{k}_{m_1}^2) \cos 2\bar{\beta} \bar{\phi} \\ - \bar{\Omega}^2 \bar{m} \bar{e} \bar{x} (-\sin \bar{\beta} \bar{v}' \cos \bar{\beta} \bar{w}') + \bar{\Omega}^2 \bar{m} \bar{e} \sin \bar{\beta} \bar{v} \\ = \bar{\omega}^2 \bar{m} \bar{k}_{m_1}^2 \bar{\phi} - \bar{\omega}^2 \bar{m} \bar{e} \sin \bar{\beta} \bar{v} + \bar{\omega}^2 \bar{m} \bar{e} \cos \bar{\beta} \bar{w} \end{aligned} \quad (66)$$

where  $\bar{k}_{m_1}^2 = k_{m_1}^2 / R^2$ ;  $\bar{k}_{m_2}^2 = k_{m_2}^2 / R^2$ ;

$$\bar{k}_m^2 = k_m^2 / R^2; \quad \bar{e} = e/R; \quad ( )' = \frac{d}{d\bar{x}}$$

$$\begin{aligned} \frac{d^2 \bar{M}_y}{d\bar{x}^2} - (\bar{T} \frac{d\bar{w}}{d\bar{x}})' - (\bar{\Omega}^2 \bar{m} \bar{e} \bar{x} \cos \bar{\beta} \bar{\phi})' \\ = \bar{\omega}^2 \bar{m} \bar{w} + \bar{\omega}^2 \bar{m} \bar{e} \cos \bar{\beta} \bar{\phi} \end{aligned} \quad (67)$$

where  $\bar{T} = \bar{\Omega}^2 \int_{\bar{x}}^1 \bar{m} \bar{x} d\bar{x}$



$$\begin{aligned}
\frac{d^2 \bar{M}_z}{d\bar{x}^2} &= (\bar{T} \frac{d\bar{v}}{d\bar{x}})' + (\bar{\Omega}^2 \bar{m} \bar{x} \sin \bar{\beta} \bar{\phi})' \\
&+ \bar{\Omega}^2 \bar{m} \sin \bar{\beta} \bar{\phi} - \bar{\Omega}^2 \bar{m} \bar{v} \\
&= \bar{\omega}^2 \bar{m} \bar{v} - \bar{\omega}^2 \bar{m} \sin \bar{\beta} \bar{\phi}
\end{aligned} \tag{68}$$

Nonuniform properties are allowed in the computer program and maximum number of stations allowed for inputting the properties are 11 for each load path member and 51 for the blade. The linear variation of properties is assumed for the intermediate locations.

## 7.2 Assumed Functions

The selected basic functions are given by Eqs. (36) and (37). These functions are nondimensionalized and differentiated with respect to  $\bar{x}$  as shown below.

### 1. Bending (Both Flapwise and Chordwise)

#### a) Deflections and Derivatives:

$$\begin{aligned}
\alpha_r(\bar{x}) &= \bar{e}_r (\sin \bar{\alpha}_r \bar{x} - \sinh \bar{\alpha}_r \bar{x}) + \\
&\bar{g}_r (\cos \bar{\alpha}_r \bar{x} - \cosh \bar{\alpha}_r \bar{x})
\end{aligned} \tag{69}$$

$$\begin{aligned}
\alpha_r'(\bar{x}) &= \bar{\alpha}_r \bar{e}_r (\cos \bar{\alpha}_r \bar{x} - \cosh \bar{\alpha}_r \bar{x}) + \\
&\bar{\alpha}_r \bar{g}_r (-\sin \bar{\alpha}_r \bar{x} - \sinh \bar{\alpha}_r \bar{x})
\end{aligned} \tag{70}$$

## b) Moments and Derivatives:

$$\begin{aligned}\bar{M}_r(\bar{x}) = & \bar{\alpha}_r^2 \bar{e}_r (-\sin \bar{\alpha}_r \bar{x} - \sinh \bar{\alpha}_r \bar{x}) + \\ & \bar{\alpha}_r^2 \bar{g}_r (-\cos \bar{\alpha}_r \bar{x} - \cosh \bar{\alpha}_r \bar{x})\end{aligned}\quad (71)$$

$$\begin{aligned}\bar{M}_r(\bar{x}) = & \bar{\alpha}_r^3 \bar{e}_r (-\cos \bar{\alpha}_r \bar{x} - \cosh \bar{\alpha}_r \bar{x}) + \\ & \bar{\alpha}_r^3 \bar{g}_r (\sin \bar{\alpha}_r \bar{x} - \sinh \bar{\alpha}_r \bar{x})\end{aligned}\quad (72)$$

where  $\bar{\alpha}_r = \alpha_r$ ,  $\bar{g}_r = g_r$ ,  $\bar{e}_r = e_r$

and are defined in Eq. (36)..

## 2. Twisting/Stretching

## a) Deflections

$$\bar{\alpha}_r(x) = \sin(\bar{\delta}_r \bar{x}) \quad (73)$$

## b) Moment/Force

$$\bar{M}_r(\bar{x}) = \bar{\delta}_r \cos(\bar{\delta}_r \bar{x}) \quad (74)$$

where  $\bar{\delta}_r = (2r-1) \frac{\pi}{2}$

The clevis compatibility functions are completed assuming  $\ell_1 = 0.9$  and the coefficients for the bending deflection functions are stored in the triple subscripted variable A(I,J,K) with subroutine titled GFCTS. The notation for the arguments is as follows:

I = Number of selected functions

J = 1 →  $a_0$

J = 2 →  $a_1$

J = 3 →  $a_2$

J = 4 →  $a_3$

where  $a_0, a_1, a_2, a_3$  are coefficients of the cubic polynomial

K = 1 → Flapwise bending, Load path 1

K = 2 → Chordwise bending, Load path 1

K = 3 → Flapwise bending, Load path 2

K = 4 → Chordwise bending, Load path 2

The coefficients for the twisting/stretching deflection functions are stored in the triple subscripted variable B(I,J,K) in the subroutine GFCTS. The notation for the arguments is as follows:

I = Number of selected function

J = 1 →  $b_0$

J = 2 →  $b_1$

where  $b_0, b_1$  are coefficients of the linear function

K = 1 → Twisting, Load path 1

K = 2 → Stretching, Load path 1

K = 3 → Twisting, Load path 2

K = 4 → Stretching, Load path 2

### 7.3 Galerkin's Equations

The Galerkin's equations to solve the eigenvalues problem are given by Eqs. (28) to (31). To avoid the numerical differentiation of several products of the nonuniform blade properties, the above equations are rewritten as shown below.

$$\begin{aligned}
 & - \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (b_{3j} M'_{yj_i} - a_{3j} D_{113} w'_{j_i} + a_{4j} D_{12} \phi'_{j_i}) w'_{k_i} dk \right\} \\
 & - \sum_{j=1}^N \int_{\ell}^R (b_{3j} M'_{yj} - a_{3j} D_{213} w'_j + a_{4j} D_{22} \phi'_j) w'_k dx \\
 & + < \sum_{j=1}^N \left[ \left\{ \sum_{i=1}^n (b_{3j} M_{yj_i} - h_{z_i} b_{ij} v_{xj_i}) \right\} - b_{3j} M_{yj} \right] w'_k >_{x=\ell} \\
 & = \omega^2 \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (D_{11} a_{3j} w_{j_i} + a_{4j} D_{13} \phi_{j_i}) w_{k_i} dx \right\} \\
 & + \omega^2 \left\{ \sum_{j=1}^N \int_{\ell}^R (D_{21} a_{3j} w_j + a_{4j} D_{23} \phi_j) w_k dx \right\} \tag{75} \\
 & - \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (b_{2j} M'_{zj_i} - a_{2j} D_{113} v'_{j_i} + a_{4j} D_{114} \phi'_{j_i}) v'_{k_i} dx \right\} \\
 & + \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (a_{4j} \bar{\Omega}^2 D_{15} \phi_{j_i} - a_{2j} \bar{\Omega}^2 D_{11} v_{j_i}) v_{k_i} dx \right\}
 \end{aligned}$$

$$\begin{aligned}
& - \sum_{j=1}^N \left\{ \int_{\ell}^R (b_{2j} M'_{zj} - a_{2j} D_{213} v'_j + a_{4j} D_{214} \phi_j) v'_k dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R (a_{4j} \bar{\Omega}^2 D_{25} \phi_j - a_{2j} \bar{\Omega}^2 D_{21i} v_j) v_k dx \right\} \\
& + < \sum_{j=1}^N \left[ \left\{ \sum_{i=1}^n (b_{2j} M_{zj_i} - h_{y_i} b_{1j} v_{xj_i}) \right\} - b_{2j} M_{zj} \right] v'_k >_{x=\ell} \\
& = \omega^2 \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (D_{11i} a_{2j} v_{j_i} - a_{4j} D_{15i} \phi_{j_i}) v_{k_i} dx \right\} \\
& + \omega^2 \sum_{j=1}^N \left\{ \int_{\ell}^R (D_{21} a_{2j} v_j - a_{4j} D_{25} \phi_j) v_k dx \right\} \tag{76}
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (-b_{4j} M'_{xj_i} - D_{12i} a_{2j} v'_{j_i} + D_{14i} a_{3j} w'_{j_i} \right. \\
& \quad \left. + a_{2j} \bar{\Omega}^2 D_{15i} v_{j_i} + a_{4j} D_{16i} \phi_{j_i}) \phi_{k_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R (-b_{4j} M'_{xj} - D_{212} a_{2j} v'_j + D_{24} a_{3j} w'_j \right. \\
& \quad \left. + a_{2j} \bar{\Omega}^2 D_{25i} v_j + a_{4j} D_{26} \phi_j) \phi_k dx \right\}
\end{aligned}$$

$$\begin{aligned}
& + < \sum_{j=1}^N \left\{ \sum_{i=1}^n [b_{4j} M_{xj_i} + h_{y_i} (-b_{3j} M'_{yj_i} + a_{3j} D_{113_i} w'_{j_i} \right. \\
& \quad + a_{4j} D_{12_i} \phi_{j_i}) - h_{z_i} (-b_{2j} M'_{zj_i} + a_{2j} D_{113_i} v'_{j_i} \\
& \quad \left. - a_{4j} D_{14_i} \phi_{j_i})] - b_{4j} M_{xj} \right\} \phi_k >_{x=\ell} \\
& = \omega^2 \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^\ell (D_{17_i} a_{4j} \phi_{j_i} + D_{13_i} a_{3j} w_{j_i} \right. \\
& \quad \left. - D_{15_i} a_{2j} v_{j_i}) \phi_{k_i} dx \right\} \\
& + \omega^2 \sum_{j=1}^N \left\{ \int_\ell^R (D_{27} a_{4j} \phi_j + D_{23} a_{3j} w_j \right. \\
& \quad \left. - D_{25} a_{2j} v_j) \phi_k dx \right\} \tag{77}
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^\ell (-b_{1j} v'_{xj_i} - a_{1j} \bar{\Omega}^2 D_{11_i} u_{j_i}) u_{k_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_\ell^R (-b_{1j} v'_{xj} - a_{1j} \bar{\Omega}^2 D_{21} u_j) u_k dx \right\}
\end{aligned}$$

$$\begin{aligned}
& + \left[ \sum_{j=1}^N \left\{ \sum_{i=1}^n (b_{1j} v_{xj_i}) - b_{1j} v_{xj} \right\} u_k \right]_{x=\ell} \\
& = \omega^2 \sum_{j=1}^N \left\{ \int_0^{\ell} D11_i a_{1j} u_{j_i} u_{k_i} dx \right\} \\
& + \omega^2 \sum_{j=1}^N \left\{ \int_{\ell}^R D21 a_{1j} u_j u_k dx \right\} \quad (78)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (a_{1j} u_{j_i} - D18_i b_{1j} v_{xj_i}) v_{xk_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R (a_{1j} u_j - D28 b_{1j} v_{xj}) v_{xk} dx \right\} = 0 \quad (79)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^{\ell} (a_{4j} \phi_{j_i} - D19_i b_{4j} M_{xj_i}) M_{xk_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_{\ell}^R (a_{4j} \phi_j - D29 b_{4j} M_{xj}) M_{xk} dx \right\} = 0 \quad (80)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_0^l (a_{2j} v_{ji}'' + D_{111i} b_{3j} M_{yj_i} - D_{110i} b_{2j} M_{zj_i}) M_{zk_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_l^R (a_{2j} v_j'' + D_{211} b_{3j} M_{yj} - D_{210} b_{2j} M_{zj}) M_{zk} dx \right\} = 0 \quad (81)
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^N \left\{ \sum_{i=1}^n \int_l^R (a_{3j} w_{ji}'' - D_{112i} b_{3j} M_{yj_i} + D_{111i} b_{2j} M_{zj_i}) M_{yk_i} dx \right\} \\
& + \sum_{j=1}^N \left\{ \int_l^R (a_{3j} w_j'' - D_{212} b_{3j} M_{yj} + D_{211} b_{2j} M_{zj}) M_{yk} dx \right\} = 0 \quad (82)
\end{aligned}$$

where

$$D_{i1} = \bar{m}$$

$$D_{i2} = \bar{\Omega}^2 \bar{m} \bar{x} \cos \bar{\beta}$$

$$D_{i3} = \bar{m} \bar{e} \cos \bar{\beta}$$

$$D_{i4} = -\bar{\Omega}^2 \bar{m} \bar{x} \sin \bar{\beta}$$

$$D_{i5} = \bar{m} \bar{e} \sin \bar{\beta}$$



$$Di6 = \bar{\Omega}^2 \bar{m} (\bar{k}_{m_2}^2 - \bar{k}_{m_1}^2) \cos 2\bar{\beta}$$

$$Di7 = \bar{m} \bar{k}_m^2$$

$$Di8 = 1/\bar{EA}$$

$$Di9 = 1/\bar{GJ}$$

$$Di10 = \bar{C}_{11} / \bar{EI}_1 \bar{EI}_2$$

$$Di11 = \bar{C}_{12} / \bar{EI}_1 \bar{EI}_2$$

$$Di12 = \bar{C}_2 / \bar{EI}_1 \bar{EI}_2$$

$$Di13 = \bar{T}$$

i = 1 → Load Path Properties

i = 2 → Blade Properties

#### 7.4 Eigenvalue Problem

Eqs. (75) to (82) can be written into a set of matrix equations of the following form for  $k = 1$  to  $N$ .

$$\begin{array}{c} [A_1] \\ \text{NX4N} \end{array} \begin{array}{c} \left\{ \begin{array}{c} a_3 \\ a_4 \\ b_1 \\ b_3 \end{array} \right\} \\ 4\text{NX1} \end{array} = \omega^2 \begin{array}{c} [B_1] \\ \text{NX2N} \end{array} \begin{array}{c} \left\{ \begin{array}{c} a_3 \\ a_4 \end{array} \right\} \\ 2\text{NX1} \end{array} \quad (83)$$

$$\begin{array}{c} [A_2] \\ \text{NX4N} \end{array} \begin{array}{c} \left\{ \begin{array}{c} a_2 \\ a_4 \\ b_1 \\ b_2 \end{array} \right\} \\ 4\text{NX1} \end{array} = \omega^2 \begin{array}{c} [B_2] \\ \text{NX2N} \end{array} \begin{array}{c} \left\{ \begin{array}{c} a_2 \\ a_4 \end{array} \right\} \\ 2\text{NX1} \end{array} \quad (84)$$

$$\begin{array}{c} [A_3] \\ \text{NX6N} \end{array} \begin{array}{c} \left\{ \begin{array}{c} a_2 \\ a_3 \\ a_4 \\ b_2 \\ b_3 \\ b_4 \end{array} \right\} \\ 6\text{NX1} \end{array} = \omega^2 \begin{array}{c} [B_3] \\ \text{NX3N} \end{array} \begin{array}{c} \left\{ \begin{array}{c} a_2 \\ a_3 \\ a_4 \end{array} \right\} \\ 3\text{NX1} \end{array} \quad (85)$$

$$\begin{matrix} [A_4] \\ NX2N \end{matrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \omega^2 \begin{matrix} [B_4] \\ NXN \end{matrix} \begin{Bmatrix} a_1 \end{Bmatrix} \quad (86)$$

$$\begin{matrix} [A_5] \\ NX2N \end{matrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad (87)$$

$$\begin{matrix} [A_6] \end{matrix} \begin{Bmatrix} a_4 \\ b_4 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad (88)$$

$$\begin{matrix} [A_7] \\ NX3N \end{matrix} \begin{Bmatrix} a_2 \\ b_2 \\ b_3 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad (89)$$

$$\begin{matrix} [A_8] \\ NX3N \end{matrix} \begin{Bmatrix} a_3 \\ b_2 \\ b_3 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad (90)$$

The above set of equations correspond to Eqs. (75) to (82) respectively.

Eq. (83) can be partitioned into the following form:

$$\begin{aligned}
 & \left| [A_{11}] \mid [A_{12}] \right| \left\{ \begin{array}{c} a_3 \\ \hline a_4 \end{array} \right\} + \left| [A_{13}] \mid [A_{14}] \right| \left\{ \begin{array}{c} b_1 \\ \hline b_3 \end{array} \right\} \\
 & = \omega^2 \left| [B_{11}] \mid [B_{12}] \right| \left\{ \begin{array}{c} a_3 \\ a_4 \end{array} \right\}
 \end{aligned} \tag{91}$$

By employing a similar partition scheme for Eqs. (84) to (86), the four equations (83 to 86) can be combined into the following form

$$\begin{aligned}
 & [E_1] \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T + [E_2] \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}^T \\
 & = \omega^2 [E_3] \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T
 \end{aligned} \tag{92}$$

where

$$\begin{array}{l} [E_1] \\ 4N \times 4N \end{array} = \begin{bmatrix} 0 & 0 & A_{11} & A_{12} \\ 0 & A_{21} & 0 & A_{22} \\ 0 & A_{31} & A_{32} & A_{33} \\ A_{41} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} [E_2] \\ 4N \times 4N \end{array} = \begin{bmatrix} A_{13} & 0 & A_{14} & 0 \\ A_{23} & A_{24} & 0 & 0 \\ 0 & A_{34} & A_{35} & A_{36} \\ A_{42} & 0 & 0 & 0 \end{bmatrix}$$

$$[E_1] = \begin{bmatrix} 0 & 0 & B_{11} & B_{12} \\ 0 & B_{21} & 0 & B_{22} \\ 0 & B_{31} & B_{32} & B_{33} \\ B_{41} & 0 & 0 & 0 \end{bmatrix}$$

Eqs. (87) to (90) can be combined into a matrix equation of the following form using a similar partition scheme employed in Eq. (91).

$$\begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}^T = [E_4] \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T \quad (93)$$

where

$$E_4 = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & c_{11} & c_{12} & 0 \\ 0 & c_{12} & c_{22} & 0 \\ 0 & 0 & 0 & d_2 \end{bmatrix}$$

$$[d_1] = -[A_{52}]^{-1} [A_{51}]$$

$$[d_2] = -[A_{62}]^{-1} [A_{61}]$$

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} A_{72} & A_{73} \\ A_{82} & A_{83} \end{bmatrix}^{-1} \begin{bmatrix} A_{71} & 0 \\ 0 & A_{81} \end{bmatrix}$$

Substituting Eq. (3) into Eq. (92) yields

$$([D] - \omega^2[I]) \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T = \begin{Bmatrix} 0 \end{Bmatrix} \quad (94)$$

where

$$[D] = [E_3]^{-1} ([E_1] + [E_2] [E_4])$$

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PROGRAM MOGALK

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DYNAMIC ANALYSIS OF MULTIPLE-LOAD-PATH BEARINGLESS ROTOR BLADES  
MODIFIED GALERKIN'S METHOD COUPLED FLAPWISE BENDING, CHORDWISE  
BENDING, TWISTING, AND STRECHING MOTION OF PRETWISTED NONUNIFORM  
ROTATING BLADES.

\*\*\*\*\*

REAL MASS, MASS1 (2, 11), MASS2 (51), KM1S1 (2, 11), KM1S2 (51), KM2S2 (51),  
1KM2S1 (2, 11)

DIMENSION STA1 (11), STA2 (51), sta (51), EIY1 (2, 11), EIY2 (51), EIZ1 (2,  
111), EIZ2 (51), EA1 (2, 11), EA2 (51), BETA1 (2, 11), BETA2 (51), GJ1 (2, 11),  
2GJ2 (51), E1 (2, 11), E2 (51), FD (2, 2), D21 (51), D22 (51), D23 (51), D24 (51),  
3D25 (51), D26 (51), D27 (51), D28 (51), D29 (51), D210 (51), D211 (51),  
4D212 (51), D213 (51), D11 (2, 11), D12 (2, 11), D13 (2, 11), D14 (2, 11),  
5D15 (2, 11), D16 (2, 11), D17 (2, 11), D18 (2, 11), D19 (2, 11), D110 (2, 11),  
6D111 (2, 11), D112 (2, 11), D113 (2, 11), BB (3), AT (2), DS (20, 20),  
7A (5, 4, 4), B (5, 2, 4), D214 (51), FRE (51)

COMPLEX DSC (20, 20), LAMDA (20), VECT (20, 20), HL (20, 20), H1 (20, 20),  
1CNT (20), COLM (20)

INTEGER INTH (20, 2)

COMMON/XM1/NPATH

COMMON/XM2/SPAN1, SPAN2, SPAN3, FD, OMEGAN

COMMON/XM3/D11, D12, D13, D14, D15, D16, D17, D18, D19, D110, D111, D112,  
1D113, D21, D22, D23, D24, D25, D26, D27, D28, D29, D210, D211, D212, D213

COMMON/XM4/STA, NS

COMMON/XM5/HS1, HS2

COMMON/XGFCT1/A, B

COMMON/XGPR1/DS



-----  
THIS SECTION READS THE DATA OF THE SYSTEM  
-----

READ (20,100) NPATH, NS1, NS2, MODES, ITT, NV

READ (20,105) SPAN1, SPAN2, OMEGA

READ (20,105) (STA1(I), I=1, NS1)

READ (20,105) (STA2(I), I=1, NS2)

DO 400 I=1, NPATH

    READ (20,105) (MASS1(I,J), J=1, NS1)

    READ (20,105) (EIY1(I,J), J=1, NS1)

    READ (20,105) (EIZ1(I,J), J=1, NS1)

    READ (20,105) (GJ1(I,J), J=1, NS1)

    READ (20,105) (E1(I,J), J=1, NS1)

    READ (20,105) (BETA1(I,J), J=1, NS1)

    READ (20,105) (KM1S1(I,J), J=1, NS1)

    READ (20,105) (KM2S1(I,J), J=1, NS1)

    READ (20,105) (EA1(I,J), J=1, NS1)

400 CONTINUE

    READ (20,105) (MASS2(I), I=1, NS2)

    READ (20,105) (EIY2(I), I=1, NS2)

    READ (20,105) (EIZ2(I), I=1, NS2)

    READ (20,105) (GJ2(I), I=1, NS2)

    READ (20,105) (E2(I), I=1, NS2)

    READ (20,105) (BETA2(I), I=1, NS2)

    READ (20,105) (KM1S2(I), I=1, NS2)

    READ (20,105) (KM2S2(I), I=1, NS2)

    READ (20,105) (EA2(I), I=1, NS2)

```
DO 401 I=1,NPATH
```

```
DO 401 J=1,2
```

```
401 FD(I,J)=0.0
```

```
IF (NPATH.GT.1) READ(20,105) ((FD(I,J),J=1,2),I=1,NPATH)
```

```
C
```

```
C
```

```
C
```

```
C
```

```
SPAN=SPAN1*SPAN2
```

```
WRITE(22,200)
```

```
WRITE(22,205)
```

```
WRITE(22,347) MODES
```

```
WRITE(22,371) NV
```

```
WRITE(22,225) SPAN
```

```
WRITE(22,230) SPAN1
```

```
WRITE(22,235) SPAN2
```

```
WRITE(22,240) OMEGA
```

```
WRITE(22,245) NS1
```

```
WRITE(22,250) NS2
```

```
WRITE(22,315) NPATH
```

```
WRITE(22,316) ITT
```

```
IF (OMEGA.LE.0) ITT=1
```

```
DO 405 I=1,NPATH
```

```
IF (I.EQ.1) WRITE(22,255)
```

```
IF (I.EQ.2) WRITE(22,260)
```

```
WRITE(22,270)
```

```
WRITE(22,275) (STA1(J),J=1,NS1)
```

```
WRITE(22,280)
```

```
WRITE(22,275) (MASS1(I,J),J=1,NS1)
```

```
WRITE(22,285)
```

```
WRITE(22,275) (EIY1(I,J),J=1,NS1)
```

```
WRITE(22,350)
```

```
WRITE(22,275) (EIZ1(I,J),J=1,NS1)
```

```
WRITE(22,352)
```

```
WRITE(22,275) (GJ1(I,J),J=1,NS1)
```

```
WRITE(22,354)
```

```
WRITE(22,275) (E1(I,J),J=1,NS1)
```

```
WRITE(22,356)
```

```
WRITE(22,275) (BETA1(I,J),J=1,NS1)
```

```
WRITE(22,358)
```

```
WRITE(22,275) (KM1S1(I,J),J=1,NS1)
```

```
WRITE(22,360)
```

```
WRITE(22,275) (KM2S1(I,J),J=1,NS1)
```

```
WRITE(22,286)
```

```
WRITE(22,275) (EA1(I,J),J=1,NS1)
```

```
IF (NPATH.GT.1) WRITE(22,321) I,FD(I,1),FD(I,2)
```

```
405 CONTINUE
```

```
WRITE(22,290)
```

```
WRITE(22,270)
```

```
WRITE(22,275) (STA2(I),I=1,NS2)
```

```
WRITE(22,280)
```

```
WRITE(22,275) (MASS2(I),I=1,NS2)
```

```
WRITE(22,285)
```

```
WRITE(22,275) (EIY2(I),I=1,NS2)
```

```
WRITE(22,350)
```

```
WRITE(22,275) (EIZ2(I),I=1,NS2)
```

```
WRITE (22,352)
WRITE (22,275) (GJ2(I),I=1,NS2)
WRITE (22,354)
WRITE (22,275) (E2(I),I=1,NS2)
WRITE (22,356)
WRITE (22,275) (BETA2(I),I=1,NS2)
WRITE (22,358)
WRITE (22,275) (KM1S2(I),I=1,NS2)
WRITE (22,360)
WRITE (22,275) (KM2S2(I),I=1,NS2)
WRITE (22,286)
WRITE (22,275) (EA2(I),I=1,NS2)
NS=NS1
DO 406 J=1,NS
  STA(J)=STA1(J)
406 CONTINUE
HH=SPAN1/10.0
DO 418 I=1,NPATH
  DO 410 J=1,NS1
    D21(J)=MASS1(I,J)
    D22(J)=EIY1(I,J)
    D23(J)=EIZ1(I,J)
    D24(J)=GJ1(I,J)
    D25(J)=E1(I,J)
    D26(J)=BETA1(I,J)
    D27(J)=KM1S1(I,J)
    D28(J)=KM2S1(I,J)
    D29(J)=EA1(I,J)
```

410 CONTINUE

CALL INTPOL(11,D21,HH)

CALL INTPOL(11,D22,HH)

CALL INTPOL(11,D23,HH)

CALL INTPOL(11,D24,HH)

CALL INTPOL(11,D25,HH)

CALL INTPOL(11,D26,HH)

CALL INTPOL(11,D27,HH)

CALL INTPOL(11,D28,HH)

CALL INTPOL(11,D29,HH)

DO 415 J=1,11

MASS1(I,J)=D21(J)

EIY1(I,J)=D22(J)

EIZ1(I,J)=D23(J)

GJ1(I,J)=D24(J)

E1(I,J)=D25(J)

BETA1(I,J)=D26(J)

KM1S1(I,J)=D27(J)

KM2S1(I,J)=D28(J)

EA1(I,J)=D29(J)

415 CONTINUE

IF (I.EQ.1) WRITE(22,255)

IF (I.EQ.2) WRITE(22,260)

WRITE(22,280)

WRITE(22,295)

WRITE(22,275) (D21(J),J=1,11)

WRITE(22,285)

WRITE(22,275) (D22(J),J=1,11)

```
WRITE (22,350)
WRITE (22,275) (D23(J),J=1,11)
WRITE (22,352)
WRITE (22,275) (D24(J),J=1,11)
WRITE (22,354)
WRITE (22,275) (D25(J),J=1,11)
WRITE (22,356)
WRITE (22,275) (D26(J),J=1,11)
WRITE (22,358)
WRITE (22,275) (D27(J),J=1,11)
WRITE (22,360)
WRITE (22,275) (D28(J),J=1,11)
WRITE (22,286)
WRITE (22,275) (D29(J),J=1,11)
```

```
418 CONTINUE
```

```
NS=NS2
DO 419 J=1,NS
STA(J)=STA2(J)
```

```
419 continue
```

```
HH=SPAN2/100.0
CALL INTPOL(51,MASS2,HH)
CALL INTPOL(51,EIY2,HH)
CALL INTPOL(51,EIZ2,HH)
CALL INTPOL(51,GJ2,HH)
CALL INTPOL(51,E2,HH)
CALL INTPOL(51,BETA2,HH)
CALL INTPOL(51,KM1S2,HH)
CALL INTPOL(51,KM2S2,HH)
```

```
CALL INTPOL(51,EA2,HH)
CALL INTPOL(51,CH,HH)
CALL INTPOL(51,QE,HH)
WRITE(22,300)
WRITE(22,280)
WRITE(22,275) (MASS2(J),J=1,51)
WRITE(22,285)
WRITE(22,275) (EIY2(J),J=1,51)
WRITE(22,350)
WRITE(22,275) (EIZ2(J),J=1,51)
WRITE(22,352)
WRITE(22,275) (GJ2(J),J=1,51)
WRITE(22,354)
WRITE(22,275) (E2(J),J=1,51)
WRITE(22,356)
WRITE(22,275) (BETA2(J),J=1,51)
WRITE(22,358)
WRITE(22,275) (KM1S2(J),J=1,51)
WRITE(22,360)
WRITE(22,275) (KM2S2(J),J=1,51)
WRITE(22,286)
WRITE(22,275) (EA2(J),J=1,51)
WRITE(22,205)
```

```
C -----
C THIS SECTION NONDIMENSIONALIZED THE DATA
C -----
C
```

```
PI=4.0*ATAN(1.0)
```

```
OMEGA=OMEGA*PI/30.0
OMEGAS=OMEGA*OMEGA
SPANS=SPAN*SPAN
EIY=EIY2(1)
MASS=MASS2(1)
FACT=SQRT(SPANS*SPANS*MASS/EIY)
OMEGAN=OMEGAS*FACT*FACT
FA=SPANS/EIY
SPAN1=SPAN1/SPAN
SPAN2=SPAN2/SPAN
HS1=SPAN1/10.0
HS2=SPAN2/50.0
SPAN3=0.7*SPAN1
DO 420 I=1,NPATH
DO 420 J=1,11
BETA1(I,J)=BETA1(I,J)*PI/180.0
KM1S1(I,J)=KM1S1(I,J)/(MASS1(I,J)*SPANS)
KM2S1(I,J)=(KM2S1(I,J)/MASS1(I,J)+E1(I,J)*E1(I,J))/SPANS
E1(I,J)=E1(I,J)/SPAN
MASS1(I,J)=MASS1(I,J)/SPAN
EIY1(I,J)=EIY1(I,J)/EIY
EIZ1(I,J)=EIZ1(I,J)/EIY
GJ1(I,J)=GJ1(I,J)/EIY
EA1(I,J)=EA1(I,J)*FA
420 CONTINUE
DO 4201 J=1,51
BETA2(J)=(BETA2(J))*PI/180.0
KM1S2(J)=KM1S2(J)/(MASS2(J)*SPANS)
```



$KM2S2(J) = (KM2S2(J) / MASS2(J) + E2(J) * E2(J)) / SPANS$

$E2(J) = E2(J) / SPAN$

$EIY2(J) = EIY2(J) / EIY$

$EIZ2(J) = EIZ2(J) / EIY$

$GJ2(J) = GJ2(J) / EIY$

$EA2(J) = FA * EA2(J)$

4201 CONTINUE

-----  
 C THIS SECTION CALCULATES THE COEFFICIENTS IN THE GALERKIN'S EQUATIONS  
 C -----  
 C

X=0.0

DO 430 I=1,NPATH

DO 430 J=1,11

TC1=OMEGA\*MASS1(I,J)

C=COS(BETA1(I,J))

S=SIN(BETA1(I,J))

D11(I,J)=MASS1(I,J)

D12(I,J)=TC1\*E1(I,J)\*X

D14(I,J)=D12(I,J)\*S

D12(I,J)=D12(I,J)\*C

D13(I,J)=MASS1(I,J)\*E1(I,J)

D15(I,J)=D13(I,J)\*S

D13(I,J)=D13(I,J)\*C

D16(I,J)=TC1\*(KM2S1(I,J)-KM1S1(I,J))\*(C\*C-S\*S)

D17(I,J)=MASS1(I,J)\*(KM2S1(I,J)+KM1S1(I,J))

D18(I,J)=1.0/EA1(I,J)

D19(I,J)=1.0/GJ1(I,J)

TC2=EIY1(I,J)\*EIZ1(I,J)

D110(I,J)=(EIY1(I,J)\*C\*C+EIZ1(I,J)\*S\*S)/TC2

D111(I,J)=(EIZ1(I,J)-EIY1(I,J))\*C\*S/TC2

D112(I,J)=(EIY1(I,J)\*S\*S+EIZ1(I,J)\*C\*C)/TC2

X=X+HS1

430 CONTINUE

X=SPAN1

DO 440 J=1,51

TC1=OMEGAN\*MASS2(J)

C=COS(BETA2(J))

S=SIN(BETA2(J))

D21(J)=MASS2(J)

D22(J)=TC1\*E2(J)\*X

D24(J)=D22(J)\*S

D22(J)=D22(J)\*C

D23(J)=MASS2(J)\*E2(J)

D25(J)=D23(J)\*S

D23(J)=D23(J)\*C

D26(J)=TC1\*(KM2S2(J)-KM1S2(J))\*(C\*C-S\*S)

D27(J)=MASS2(J)\*(KM2S2(J)+KM1S2(J))

D28(J)=1.0/EA2(J)

D29(J)=1.0/GJ2(J)

TC2=EIY2(J)\*EIZ2(J)

D210(J)=(EIY2(J)\*C\*C+EIZ2(J)\*S\*S)/TC2

D211(J)=(EIZ2(J)-EIY2(J))\*C\*S/TC2

D212(J)=(EIY2(J)\*S\*S+EIZ2(J)\*C\*C)/TC2

X=X+HS2

440 CONTINUE

```
X=SPAN1
H=HS2/24.0
DO 450 I=1,51
D214(I)=MASS2(I)*H*X
X=X+HS2
450 CONTINUE
D213(51)=0.0
D213(50)=(9.0*D214(51)+19.0*D214(50)-5.0*D214(49)+D214(48))
1*OMEGAN
DO 455 I=2,49
J=51-I
D213(J)=D213(J+1)+(-D214(J+2)+13.0*(D214(J+1)+D214(J))-
1D214(J-1))*OMEGAN
455 CONTINUE
D213(1)=D213(2)+(D214(4)-5.0*D214(3)+19.0*D214(2)+9.0*D214(1))
1*OMEGAN
GO TO (460,465),NPATH
460 BB(1)=D213(1)
GO TO 480
465 H=SPAN1/24.0
DO 475 J=1,NPATH
AT(J)=(9.0*D18(J,1)+19.0*D18(J,2)-5.0*D18(J,3)+D18(J,4))*H
DO 470 K=1,9
AT(J)=AT(J)+H*(-D18(J,K)+13.0*D18(J,K+1)+13.0*D18(J,K+2)
1-D18(J,K+3))
470 CONTINUE
AT(J)=AT(J)+H*(D18(J,8)-5.0*D18(J,9)+19.0*D18(J,10)+9.0*D18(J,11))
475 CONTINUE
```

```

      BB(1)=AT(1)*D213(1)/(AT(1)+AT(2))
      BB(2)=AT(2)*D213(1)/(AT(1)+AT(2))
480 DO 495 JJ=1,NPATH
      X=0.0
      DO 485 I=1,11
        D214(I)=MASS1(JJ,I)*H*X
        X=X+HS1
485 CONTINUE
      D113(JJ,11)=0.0
      D113(JJ,10)=(9.0*D214(11)+19.0*D214(10)-5.0*D214(9)+D214(8))
      1*OMEGAN
      DO 490 I=2,9
        J=11-I
        D113(JJ,J)=D113(JJ,J+1)+C-D214(J+2)+13.0*(D214(J+1)+D214(J)
      1-D214(J-1))*OMEGAN
490 CONTINUE
      D113(JJ,1)=D113(JJ,2)+(D214(4)-5.0*D214(3)+19.0*D214(2)
      1+9.0*D214(1))*OMEGAN
495 CONTINUE
      DO 500 I=1,NPATH
        DO 500 J=1,11
500 D113(I,J)=D113(I,J)+BB(I)
      IF (NPATH.GT.1.AND.ITT.GT.1) CALL TITER(ITT)

```

```

C -----
C THIS SECTION CALCULATES GALERKIN'S FUNCTIONS AND WRITES THE
C COEFFICIENTS OF THE CLEVIS COMPATIBILITY POLYNOMIALS
C -----
C

```

```
CALL GFCTS(MODES)
```

```
N1=2*NPATH
```

```
WRITE(22,361)
```

```
DO 505 K=1,MODES
```

```
WRITE(22,366)
```

```
DO 505 I=1,N1,2
```

```
IF (I.EQ.1) WRITE(22,362)
```

```
IF (I.EQ.2) WRITE(22,363)
```

```
WRITE(22,364)
```

```
WRITE(22,275) (A(K,J,I),J=1,4)
```

```
WRITE(22,365)
```

```
WRITE(22,275) (A(K,J,I+1),J=1,4)
```

```
WRITE(22,367)
```

```
WRITE(22,275) (B(K,J,I),J=1,2)
```

```
WRITE(22,368)
```

```
WRITE(22,275) (B(K,J,I+1),J=1,2)
```

```
505 CONTINUE
```

```
-----  
C THIS SECTION CALCULATES THE EIGENVALUES AND EIGENVECTORS  
C -----  
C
```

```
N4=4*NPATH
```

```
DO 510 I=1,N4
```

```
DO 510 J=1,N4
```

```
510 DSC(I,J)=CMPLX(DS(I,J),0.0)
```

```
INTH(1,1)=N4
```

```
INTH(2,1)=NV
```

```
CALL EECM(DSC,LAMDA,VECT,HL,H1,CNT,COLM,INTH)
```

```
WRITE(22,200)
WRITE(22,205)
WRITE(22,369)
WRITE(22,205)
WRITE(22,275) (LAMDA(J),J=1,N4)
DO 515 J=1,N4
515 FRE(J)=SQRT(REAL(LAMDA(J)))
WRITE(22,370)
WRITE(22,275) (FRE(J),J=1,N4)
IF (NV.EQ.0) GO TO 600
WRITE(22,371)
DO 520 I=1,N4
520 WRITE(22,275) (VECT(I,J),J=1,NV)
WRITE(22,372)
DO 530 J=1,NV
AMAX=REAL(VECT(I,J))
DO 525 I=2,N4
525 IF (ABS(AMAX).LT.ABS(REAL(VECT(I,J)))) AMAX=REAL(VECT(I,J))
DO 530 I=1,N4
530 VECT(I,J)=VECT(I,J)/AMAX
DO 535 I=N4
535 WRITE(22,275) (VECT(I,J),J=1,NV)
WRITE(22,205)
```

C

C

FORMATS

C

C

```

100 FORMAT(5I5)
105 FORMAT(5E14.7)
200 FORMAT(1H1)
205 FORMAT(/2X,'*****',
      $'*****')
225 FORMAT(/5X,'RADIUS OF THE ROTOR (INCHES)    =',E14.7)
230 FORMAT(/5X,'LENGTH OFF INBOARD SEGMENTS (IN)=' ,E14.7)
235 FORMAT(/5X,'LENGTH OF THE BLADE (INCHES)    =',E14.7)
240 FORMAT(/5X,'ROTATIONAL SPEED (RPM)          =',E14.7)
245 FORMAT(/5X,'NUMBER OF DATA POINTS FOR INBOARD SEGMENTS=' ,I5)
250 FORMAT(/5X,'NUMBER OF DATA POINTS FOR BLADE          =' ,I5)
255 FORMAT(/5X,'PROPERTIES OF THE FIRST LOAD PATH')
260 FORMAT(/5X,'PROPERTIES OF THE SECOND LOAD PATH')
270 FORMAT(/5X,'DATA POINT LOCATIONS IN INCHES')
275 FORMAT(4(6X,E14.7))
280 FORMAT(/5X,'MASS PER UNIT LENGTH (LB-SEC.**2/IN**2)')
285 FORMAT(/5X,'FLAPWISE BENDING STIFFNESS (LB-IN**2)')
286 FORMAT(/5X,'AXIAL STIFFNESS(LB) ')
290 FORMAT(/5X,'PROPERTIES OF THE BLADE')
295 FORMAT(/5X,'INTERPOLATED VALUES FOR INBOARD SEGMENTS, 11',
      $'EQUIDISTANT VALUES')
300 FORMAT(/5X,'INTERPOLATED VALUES FOR THE BLADE, 51',
      $'EQUIDISTANT VALUES')
315 FORMAT(/5X,'MUNBER OF LOAD PATHS')
316 FORMAT(/5X,'NUMBER OF FLNSION ITERATIONS =' ,I5)
321 FORMAT(/5X,'Y,Z DISTANCES BETWEEN LOADPATH NO:',I2,
      $/5X,' AND THE BLADE(IN) ARE=' ,F6.2,5X,F6.2)
347 FORMAT(/5X,'NUMBER OF MODES TO BE USED =' ,I5)

```

```

350 FORMAT(//5X,'CHORDWISE BENDING STIFFNESS(LB-IN**2)')
352 FORMAT(//5X,'TORSIONAL STIFFNESS(LB-IN**2)')
354 FORMAT(//5X,'DISTANCE BETWEEN MASS AND ELASTIC AXIS(IN)=')
356 FORMAT(//5X,'TWIST OF THE BLADE',/5X,'(DEGREES) =')
358 FORMAT(//5X,'MASS MOMENT OF INERTIA ABOUT THE CHORD(LB-SEC**2)')
360 FORMAT(//5X,'MASS MOMENT OF INERTIA ABOUT AN AXIS',
    $/5X,'PERPINDICULAR TO THE CHORD THROUGH THE CENTER OF',
    $/5X,'GRAVITY(LB-SEC**2)')
361 FORMAT(//5X,'THE COEFFICIENTS OF THE CLEVIS COMPATIBILITY
    $FUNCTIONS ARE')
362 FORMAT(//5X,'COEFFICIENTS OF THE FIRST LOAD PATH')
363 FORMAT(//5X,'COEFFICIENTS OF THE SECOND LOAD PATH')
364 FORMAT(//5X,'FLAPWISE BENDING')
365 FORMAT(//5X,'CHORDWISE BENDING')
366 FORMAT(//5X,'MODE NUMBER')
367 FORMAT(//5X,'TWISTING')
368 FORMAT(//5X,'STRETCHING')
369 FORMAT(//5X,'EIGENVALUES ARE AS FOLLOWS')
370 FORMAT(//5X,'NATURAL FREQUENCIES ARE AS FOLLOWS')
371 FORMAT(//5X,'EIGENVECTORS ARE AS FOLLOWS')
372 FORMAT(//5X,'NORMALIZED EIGENVECTORS')

```

```

C
600 STOP

```

```

END

```

```

C *****
SUBROUTINE EECM(A,LAMDA,VECT,HL,H,CNT,COLM,INTH)
C *****

```



```

-----
COMPUTES EIGENVALUES AND EIGENVECTORS OF A COMPLEX MATRIX
-----

```

```

COMPLEX A(20,20), LAMDA(20), SHIFT(3), TEMP, TEMP1, TEMP2, SINN, COSS,
$HL(60,60), CNT(20), VECT(20,20), COLM(20), H(60,60)

```

```

LOGICAL TWICE, CON

```

```

INTEGER INTH(20,2), R, RP1, RP2

```

```

N=INTH(1,1)

```

```

M=N

```

```

NV=INTH(2,1)

```

```

DO 1 I=1,M

```

```

DO 1 J=1,M

```

```

1 HL(J,I)=A(J,I)

```

```

NCAL=N

```

```

SHIFT(1)=(0.0,0.0)

```

```

ICOUNT=0

```

```

IF (N-2) 2,37,3

```

```

2 LAMDA(1)=A(1,1)

```

```

VECT(1,1)=(1.0,1.0)

```

```

GO TO 63

```

```

C
C REDUCE MATRIX TO HESSENBURG GORM

```

```

C
3 NM2=N-2

```

```

DO 14 R=1,NM2

```

```

RP1=R+1

```

```

RP2=R+2

```

```
ABIG=0.0
INTH(R,1)=RP1
DO 4 I=RP1,N
ABSSQ=REAL(HL(I,R))**2+AIMAG(HL(I,R))**2
IF (ABSSQ.LE.ABIG) GO TO 4
INTH(R,1)=I
ABIG=ABSSQ
4 CONTINUE
INTER=INTH(R,1)
IF (INTER.EQ.0.0) GO TO 14
IF (INTER.EQ.RP1) GO TO 7
DO 5 I=R,N
TEMP=HL(RP1,I)
HL(RP1,I)=HL(INTER,I)
5 HL(INTER,I)=TEMP
DO 6 I=1,N
TEMP=HL(I,RP1)
HL(I,RP1)=HL(I,INTER)
6 HL(I,INTER)=TEMP
7 DO 8 I=RP2,N
COLM(I)=HL(I,R)/HL(RP1,R)
8 HL(I,R)=COLM(I)
DO 10 I=1,RP1
TEMP=(0.0,0.0)
DO 9 J=RP2,N
9 TEMP=TEMP+HL(I,J)*COLM(J)
10 HL(I,RP1)=HL(I,RP1)+TEMP
DO 12 I=RP2,N
```

```
TEMP=(0.0,0.0)
DO 11 J=RP2,N
11 TEMP=TEMP+HL(I,J)*COLM(J)
12 HL(I,RP1)=HL(I,RP1)+TEMP-COLM(I)*HL(RP1,RP1)
DO 13 I=RP2,N
DO 13 J=RP2,N
13 HL(I,J)=HL(I,J)-COLM(I)*HL(RP1,J)
14 CONTINUE
```

C

C

CALCULATE EPSILON

C

```
EPS=0.0
DO 15 I=1,N
15 EPS=EPS+CABS(HL(1,I))
DO 17 I=2,N
SUM=0.0
IM1=I-1
DO 16 J=IM1,N
16 SUM=SUM+CABS(HL(I,J))
IF (SUM.GT.EPS) EPS=SUM
17 CONTINUE
FN=N
EPS=SQRT(FN)*EPS*1.0E-12
IF (EPS.LT.1.0E-12) EPS=1.0E-12
DO 18 I=1,N
DO 18 J=1,N
18 H(J,I)=HL(J,I)
19 IF (N-2) 20,37,21
```

20 LAMDA(M)=HL(1,1)+SHIFT(1)

GO TO 39

21 MN1=M-N+1

CON=.FALSE.

IF (REAL(HL(N,N)).NE.0.0.OR.AIMAG(HL(N,N)).NE.0.0) CON=.TRUE.

IF (.NOT.CON) GO TO 22

HL(N,N)=HL(N,N)

IF (ABS(REAL(HL(N,N-1)/HL(N,N)))+ABS(AIMAG(HL(N,N-1)/HL  
\$(N,N)))-1.0E-12) 23,23,22

22 IF (ABS(REAL(HL(N,N-1)))+ABS(AIMAG(HL(N,N-1))).GE.EPS) GO TO 24

23 LAMDA(MN1)=HL(N,N)+SHIFT(1)

ICOUNT=0

N=N-1

GO TO 19

C

C

DETERMINE SHIFT

C

24 TEMP=HL(N,N)\*HL(N-1,N-1)-HL(N,N-1)\*HL(N-1,N)

TEMP1=HL(N-1,N-1)+HL(N,N)

SHIFT(2)=(TEMP1+CSQRT((TEMP1)\*\*2-4.0\*(TEMP)))/2.0

IF (SHIFT(2).NE.(0.0,0.0)) GO TO 25

SHIFT(3)=TEMP1

GO TO 26

25 SHIFT(3)=TEMP/SHIFT(2)

26 IF (CABS(SHIFT(2)-HL(N,N)).LT.CABS(SHIFT(3)-HL(N,N))) GO TO 27

INDEX=3

GO TO 28

27 INDEX=2

28 IF (CABS(HL(N-1,N-2)).GE.EPS) GO TO 29

LAMDA(MN1)=SHIFT(2)+SHIFT(1)

LAMDA(MN1+1)=SHIFT(3)+SHIFT(1)

ICOUNT=0

N=N-2

GO TO 19

29 SHIFT(1)=SHIFT(1)+SHIFT(INDEX)

C  
C PERFORM GIVEN ROTATIONS, QR ITERATIONS

C  
DO 30 I=1,N

30 HL(I,I)=HL(I,I)-SHIFT(INDEX)

IF (ICOUNT.LE.40) GO TO 31

NCAL=M-N

GO TO 39

31 NM1=N-1

TEMP1=HL(1,1)

TEMP2=HL(2,1)

DO 36 R=1,NM1

RP1=R+1

SUM=SQRT(REAL(TEMP1)\*\*2+AIMAG(TEMP1)\*\*2+REAL(TEMP2)\*\*2+AIMAG

\$(TEMP2)\*\*2)

IF (SUM.NE.0.0) GO TO 32

TEMP1=HL(RP1,RP1)

TEMP2=HL(R+2,RP1)

GO TO 36

32 COSS=TEMP1/SUM

SINN=TEMP2/SUM

```
INDEX=MAX0 (R-1,1)
DO 33 I=INDEX,N
TEMP=CONJG (COSS) *HL (R,I)+CONJG (SINN) *HL (RP1,I)
HL (RP1,I)=-SINN*HL (R,I)+COSS*HL (RP1,I)
33 HL (R,I)=TEMP
TEMP1=HL (RP1,RP1)
TEMP2=HL (R+2,RP1)
DO 34 I=1,R
TEMP=COSS*HL (I,R)+SINN*HL (I,RP1)
HL (I,RP1)=-CONJG (SINN) *HL (I,R)+CONJG (COSS) *HL (I,RP1)
34 HL (I,R)=TEMP
INDEX=MIN0 (R+2,N)
DO 35 I=RP1,INDEX
HL (I,R)=SINN*HL (I,RP1)
35 HL (I,RP1)=CONJG (COSS) *HL (I,RP1)
36 CONTINUE
ICOUNT=ICOUNT+1
GO TO 21
37 TEMP1=HL (N-1,N-1)+HL (N,N)
TEMP2=HL (N,N) *HL (N-1,N-1)-HL (N,N-1) *HL (N-1,N)
TEMP=(TEMP1+CSQRT ((TEMP1)**2-4.0*(TEMP2)))/2.0
IF (REAL (TEMP) .NE. 0.0) GO TO 38
LAMDA (M)=SHIFT (1)
LAMDA (M-1)=TEMP1+SHIFT (1)
GO TO 39
38 LAMDA (M)=TEMP+SHIFT (1)
LAMDA (M-1)=TEMP2/TEMP+SHIFT (1)
```

C VALUSE IN DECENDING ORDER OF ABSOLUTE MAGITUDE  
C

39 CONTINUE

IF (M.NE.2) GO TO 40

EPS=AMAX1 (CABS (LAMDA (1)), CABS (LAMDA (2))) \*1.0E-12

IF (EPS.LT.1.0E-12) EPS=1.0E-12

H (1,1)=HL (1,1)

H (1,2)=HL (1,2)

H (2,1)=HL (2,1)

H (2,2)=HL (2,2)

40 CONTINUE

IF (NCAL.LE.1) GO TO 143

K=NCAL+1

DO 41 I=1,NCAL

K=K-1

CNT (I)=LAMDA (K)

41 CONTINUE

L=NCAL-1

DO 43 I=1,L

K=I+1

DO 42 J=K,NCAL

IF (CABS (CNT (I)) .GE. CABS (CNT (J))) GO TO 42

TEMP=CNT (J)

CNT (J)=CNT (I)

CNT (I)=TEMP

42 CONTINUE

43 LAMDA (I)=CNT (I)

LAMDA (NCAL)=CNT (NCAL)

C  
C CALCULATES VECTORS  
C

143 CONTINUE

NC=MIN0(NV,NCAL)

IF(NC.EQ.0) GO TO 63

NM1=M-1

DO 62 L=1,NC

DO 45 I=1,M

DO 44 J=1,M

44 HL(J,I)=H(J,I)

45 HL(I,I)=HL(I,I)-LAMDA(L)

DO 49 I=1,NM1

COLM(I)=(0.0,0.0)

INTH(I,2)=0

IP1=I+1

IF (CABS(HL(I+1,I)).LE.CABS(HL(I,I))) GO TO 47

INTH(I,2)=I

DO 46 J=1,M

TEMP=HL(IP1,J)

HL(IP1,J)=HL(I,J)

46 HL(I,J)=TEMP

47 IF(HL(I,I).EQ.(0.0,0.0)) GO TO 49

COLM(I)=-HL(IP1,I)/HL(I,I)

DO 48 J=IP1,M

48 HL(IP1,J)=HL(IP1,J)+COLM(I)\*HL(I,J)

49 CONTINUE

DO 50 I=1,M



```
50 CNT(I)=(1.0,0.0)
    TWICE=.FALSE.
51 IF (HL(M,M).EQ.(0.0,0.0)) HL(M,M)=EPS
    CNT(M)=CNT(M)/HL(M,M)
    DO 53 I=1,NM1
        K=M-1
        DO 52 J=K,NM1
52 CNT(K)=CNT(K)-HL(K,J+1)*CNT(J+1)
        IF (HL(K,K).EQ.(0.0,0.0)) HL(K,K)=EPS
53 CNT(K)=CNT(K)/HL(K,K)
    ABIG=0.0
    DO 54 I=1,N
        SUM=ABS(REAL(CNT(I)))+ABS(AIMAG(CNT(I)))
        IF(SUM.GT.ABIG) ABIG=SUM
54 CONTINUE
    DO 55 I=1,M
55 CNT(I)=CNT(I)/ABIG
    IF(TWICE) GO TO 57
    DO 56 I=1,NM1
        IF(INTH(I,2).EQ.0) GO TO 56
        TEMP=CNT(I)
        CNT(I)=CNT(I+1)
        CNT(I+1)=TEMP
56 CNT(I+1)=CNT(I+1)+COLM(I)*CNT(I)
    TWICE=.TRUE.
    GO TO 51
57 IF(M.EQ.2) GO TO 60
    NM2=M-2
```

DO 59 I=1,NM2

N1I=M-1-I

NI1=M-I+1

DO 58 J=NI1,M

58 CNT(J)=H(J,N1I)\*CNT(N1I+1)+CNT(J)

INDEX=INTH(N1I,1)

TEMP=CNT(N1I+1)

CNT(N1I+1)=CNT(INDEX)

59 CNT(INDEX)=TEMP

C

C

NORMALIZE VECTORS TO UNIT LENGTH

C

60 CONTINUE

TEMP=(0.0,0.0)

DO 61 I=1,M

61 TEMP=TEMP+CNT(I)\*CNT(I)

TEMP=CSQRT(TEMP)

IF(TEMP.EQ.(0.0,0.0)) TEMP=(1.0,0.0)

DO 62 I=1,M

VECT(I,L)=CNT(I)/TEMP

62 CONTINUE

63 CONTINUE

INTH(1,1)=NCAL

RETURN

END

C

\*\*\*\*\*

SUBROUTINE GFCTS(M)

C

\*\*\*\*\*

-----  
 THIS SUBROUTINE CALCULATES THE GALERKIN'S FUNCTIONS  
 -----

REAL MYL(5,2,11),MYLP(5,2,11),MZL(5,2,11),MZLP(5,2,11),MXL(5,2,11)  
 1,MXLP(5,2,11),MY(5,51),MYP(5,51),MZ(5,51),MZP(5,51),MX(5,51),  
 2MXP(5,51)

DIMENSION AL(5),WL(5,2,11),WLP(5,2,11),VL(5,2,11),VLP(5,2,11),  
 1TL(5,2,11),VXL(5,2,11),VXLP(5,2,11),W(5,51),WP(5,51),V(5,51),  
 2VP(5,51),T(5,51),U(5,51),VX(5,51),VXP(5,51),C(4,4),CT(4,4),  
 3B(5,2,4),CT1(4,4),A(5,4,4),FD(2,2),ul(5,2,11)

DATA AL/1.875,4.694,7.855,10.996,14.137/

COMMON/XM1/NPATH

COMMON/XM2/SPAN1,SPAN2,SPAN3,FD,OMEGAN

COMMON/XGFCT1/A,B

COMMON/XGFCT2/MYL,MYLP,MZL,MZLP,MXL,MXLP,MY,MYP,MZ,MZP,MX,MXP,WL,  
 1WLP,VL,VLP,TL,UL,VXL,VXLP,W,WP,V,VP,T,U,VX,VXP

DO 220 I=1,M

X=0.0

SN=SIN(AL(I))

SH=SINH(AL(I))

CN=COS(AL(I))

CH=COSH(AL(I))

E=(-SN+SH)/2.0\*SN\*SH

G=(CN+CH)/2.0\*SN\*SH

D=(2.0\*FLOAT(I)-1.0)\*PI/2.0

DO 210 J=1,11

X1=X\*D

```
X=X*AL(I)
SN=SIN(X)
SH=SINH(X)
CN=COS(X)
CH=COSH(X)
CN1=COS(X1)
SN1=SIN(X1)
DO 200 K=1,NPATH
WL(I,K,J)=E*(SN-SH)+G*(CN-CH)
WLP(I,K,J)=AL(I)*E*(CN-CH)+AL(I)*G*(-SN-SH)
MYL(I,K,J)=AL(I)**2*E*(-SN-SH)+AL(I)**2*G*(-CN-CH)
MYLP(I,K,J)=AL(I)**3*E*(-CN-CH)+AL(I)**3*G*(SN-SH)
VL(I,K,J)=WL(I,K,J)
VLP(I,K,J)=WLP(I,K,J)
MZL(I,K,J)=MYL(I,K,J)
MZLP(I,K,J)=MYLP(I,K,J)
TL(I,K,J)=SN1
MXL(I,K,J)=D*CN1
MXLP(I,K,J)=-D*D*SN1
UL(I,K,J)=TL(I,K,J)
VXL(I,K,J)=MXL(I,K,J)
VXLP(I,K,J)=MXLP(I,K,J)
200 CONTINUE
X=X+HS1
210 CONTINUE
X=SPAN1
DO 220 J=1,51
X1=X*D
```

```
X=AL(I)
SN=SIN(X)
SH=SINH(X)
CN=COS(X)
CH=COSH(X)
CN1=COS(X1)
SN1=SIN(X1)
W(I,J)=E*(SN-SH)+G*(CN-CH)
WP(I,J)=AL(I)*E*(CN-CH)+AL(I)*G*(-SN-SH)
MY(I,J)=AL(I)**2*E*(-SN-SH)+AL(I)**2*G*(-CN-CH)
MYP(I,J)=AL(I)**3*E*(-CN-CH)+AL(I)**3*G*(SN-SH)
V(I,J)=W(I,J)
VP(I,J)=WP(I,J)
MZ(I,J)=MYP(I,J)
T(I,J)=SN1
MX(I,J)=D*CN1
MXP(I,J)=-D*D*SN1
U(I,J)=T(I,J)
VX(I,J)=MX(I,J)
VXP(I,J)=MXP(I,J)
X=X+HS2
```

```
220 CONTINUE
```

```
C(1,1)=1.0
C(1,2)=SPAN3
C(1,3)=SPAN3*SPAN3
C(1,4)=C(1,3)*SPAN3
C(2,1)=0.0
C(2,2)=1.0
```

```
C(2,3)=2.0*SPAN3
C(2,4)=3.0*SPAN3*SPAN3
C(3,1)=1.0
C(3,2)=SPAN1
C(3,3)=SPAN1*SPAN1
C(3,4)=C(3,3)*SPAN1
C(4,1)=0.0
C(4,2)=1.0
C(4,3)=2.0*SPAN1
C(4,4)=3.0*SPAN1*SPAN1
CALL SOLUTN(C,4,-1,4)
N=NPATH+1
n2=2*npath
DO 240 I=1,M
DO 230 K=1,N2,2
K1=K
IF(K.GT.1) K1=K-1
CT(1,K)=WL(I,K1,10)
CT(2,K)=WLP(I,K1,10)
CT(3,K)=W(I,1)+FD(K1,1)*T(I,1)
CT(4,K)=WP(I,1)
CT(1,K+1)=VL(I,K1,10)
CT(2,K+1)=VLP(I,K1,10)
CT(3,K+1)=V(I,1)-FD(K1,2)*T(I,1)
CT(4,K+1)=VP(I,1)
B(I,1,K)=(SPAN1*TL(I,K1,10)-SPAN3*T(I,1))/(SPAN1-SPAN3)
B(I,2,K)=(T(I,1)-TL(I,K1,10))/(SPAN1-SPAN3)
B(I,1,K+1)=(SPAN1*UL(I,K1,10)-SPAN3*U(I,1))/(SPAN1-SPAN3)
```

$B(I, 2, K+1) = (U(I, 1) - UL(I, K1, 10)) / (SPAN1 - SPAN3)$

230 CONTINUE

CALL MATMUL(C, CT, CT1, 4, 4, N2)

DO 240 II=1, 4

DO 240 IJ=1, N2

A(I, II, IJ) = CT1(II, IJ)

240 CONTINUE

DO 260 I=1, MODES

DO 260 J=1, NPATH

IJ=2\*J-1

TL(I, J, 11) = 0.0

UL(I, J, 11) = 0.0

WL(I, J, 11) = 0.0

VL(I, J, 11) = 0.0

WLP(I, J, 11) = 0.0

VLP(I, J, 11) = 0.0

WL(I, J, 11) = A(I, L, IJ) \* SPAN1 \*\* (L-1) + WL(I, J, 11)

VL(I, J, 11) = A(I, L, IJ+1) \* SPAN1 \*\* (L-1) + VL(I, J, 11)

WLP(I, J, 11) = FLOAT(L-1) \* A(I, L, IJ) \* SPAN1 \*\* (L-2) + WLP(I, J, 11)

VLP(I, J, 11) = FLOAT(L-1) \* A(I, L, IJ+1) \* SPAN1 \*\* (L-2) + VLP(I, J, 11)

250 CONTINUE

TL(I, J, 11) = B(I, 1, IJ) + B(I, 2, IJ) \* SPAN1

UL(I, J, 11) = B(I, 1, IJ+1) + B(I, 2, IJ+1) \* SPAN1

260 CONTINUE

RETURN

END

C

\*\*\*\*\*

SUBROUTINE GPROC(MODES)

\*\*\*\*\*

-----  
THIS SUBROUTINE FORMULATES THE EIGENVALUE PROBLEM  
-----

REAL INTG

DIMENSION A11(5,5),A12(5,5),A21(5,5),A22(5,5),A31(5,5),A32(5,5),  
1A33(5,5),A41(5,5),A13(5,5),A14(5,5),A23(5,5),A24(5,5),A34(5,5),  
2A35(5,5),A36(5,5),A42(5,5),B11(5,5),B12(5,5),B21(5,5),b33(5,5),  
3B22(5,5),B31(5,5),B32(5,5),B41(5,5),A51(5,5),A52(5,5),A61(5,5),  
4A62(5,5),A71(5,5),A72(5,5),A73(5,5),A83(5,5),D11(2,11),D12(2,11),  
5D13(2,11),D14(2,11),D15(2,11),D16(2,11),D17(2,11),D18(2,11),  
6D19(2,11),D110(2,11),D21(51),D22(51),D23(51),d111(2,11),  
7D24(51),D25(51),D26(51),D27(51),D28(51),D29(51),D210(51),D211(51),  
8D212(51),F11(2,11),F12(2,11),F13(2,11),F14(2,11),F15(2,11),F21  
9(51),F22(51),F23(51),F24(51),F25(51),FD(2,2),F16(2,11),F17(2,11),  
1F26(51),F27(51),D1(5,5),D2(5,5),C(10,10),C1(10,10),C2(10,10),  
2WL(5,2,11),WLP(5,2,11),VL(5,2,11),VLP(5,2,11),TL(5,2,11),UL(5,2,  
311),VXL(5,2,11),VXLP(5,2,11),W(5,51),WP(5,51),V(5,51),VP(5,51),  
4t(5,51),U(5,51),VX(5,51),VXP(5,51),A(5,4,4),B(5,2,4),E1(20,20),  
5e2(20,20),E3(20,20),E4(20,20),DS(20,20),d112(2,11),d113(2,11),  
6d213(51),a81(5,5),a82(5,5)

REAL MYL(5,2,11),MYLP(5,2,11),MZL(5,2,11),MZLP(5,2,11),  
1mx1(5,2,11),MXLP(5,2,11),MY(5,51),MYP(5,51),MZ(5,51),MZP(5,51),  
2mx(5,51),MXP(5,51)

COMMON/XM1/NPATH

COMMON/XM2/SPAN1,SPAN2,SPAN3,FD,OMEGAN

COMMON/XM3/D11,D12,D13,D14,D15,D16,D17,D18,D19,D110,D111,D112,D113  
1,D21,D22,D23,D24,D25,D26,D27,D28,D29,D210,D211,D212,D213



```
COMMON/XGFCT2/MYL,MYLP,MZL,MZLP,MXL,MXLP,MY,MYP,MZ,MZP,MX,MXP,WL,
1WLP,VL,VLP,TL,UL,VXL,VXLP,W,WP,V,VP,T,U,VX,VXP
```

```
COMMON/XGPR1/DS
```

```
DO 120 K=1,MODES
```

```
DO 120 J=1,MODES
```

```
DO 100 I=1,NPATH
```

```
DO 100 L=1,11
```

```
F11(I,L)=D113(I,L)*WLP(J,I,L)*WLP(K,I,L)
```

```
F12(I,L)=D12(I,L)*TL(J,I,L)*WLP(K,I,L)
```

```
F13(I,L)=-MYLP(J,I,L)*WLP(K,I,L)
```

```
F14(I,L)=D11(I,L)*WL(J,I,L)*WL(K,I,L)
```

```
F15(I,L)=D13(I,L)*TL(J,I,L)*WL(K,I,L)
```

```
100 CONTINUE
```

```
DO 110 L=1,51
```

```
F21(L)=D213(L)*WP(J,L)*WP(K,L)
```

```
F22(L)=D22(L)*T(J,L)*WP(K,L)
```

```
F23(L)=-MYP(J,L)*WP(K,L)
```

```
F24(L)=D21(L)*W(J,L)*W(K,L)
```

```
F25(L)=D23(L)*T(J,L)*W(K,L)
```

```
110 CONTINUE
```

```
A11(K,J)=INTG(F11,F12)
```

```
A12(K,J)=INTG(F12,F22)
```

```
A14(K,J)=INTG(F13,F23)
```

```
B11(K,J)=INTG(F14,F24)
```

```
B12(K,J)=INTG(F15,F25)
```

```
A14(K,J)=A14(K,J)+W(K,1)*(MYL(J,1,11)+MYL(J,2,11)-MY(J,1))
```

```
A13(K,J)=-FD(1,2)*VXL(J,1,11)-FD(2,2)*VXL(J,2,11)
```

```
120 CONTINUE
```

```
DO 150 K=1,MODES
DO 150 J=1,MODES
DO 130 I=1,NPATH
DO 130 L=1,11
F11(I,L)=D113(I,L)*VLP(J,I,L)*VLP(K,I,L)
F12(I,L)=D14(I,L)*TL(J,I,L)*VLP(K,I,L)
F13(I,L)=-MZLP(J,I,L)*VLP(K,I,L)
F14(I,L)=D11(I,L)*VL(J,I,L)*VL(K,I,L)
F15(I,L)=-D15(I,L)*TL(J,I,L)*VL(K,I,L)
130 CONTINUE
DO 140 L=1,51
F21(L)=D213(L)*VP(J,L)*VP(K,L)
F22(L)=D24(L)*T(J,L)*VP(K,L)
F23(L)=-MZP(J,L)*VP(K,L)
F24(L)=D21(L)*V(J,L)*V(K,L)
F25(L)=-D25(L)*T(J,L)*V(K,L)
140 CONTINUE
A21(K,J)=INTG(F11,F21)
A22(K,J)=INTG(F12,F22)
A24(K,J)=INTG(F13,F23)
B21(K,J)=INTG(F14,F24)
B22(K,J)=INTG(F15,F25)
A24(K,J)=A24(K,J)+V(K,1)*(MZL(J,1,11)+MZL(J,2,11)-MZ(J,1))
A23(K,J)=-FD(1,1)*VXL(J,1,11)-FD(2,1)*VXL(J,2,11)
150 CONTINUE
DO 180 K=1,MODES
DO 180 J=1,MODES
DO 160 I=1,NPATH
```

```

DO 160 L=1,11
  F11(I,L)=-D12(I,L)*VLP(J,I,L)*TL(K,I,L)+OMEGAN*D15(I,L)*
1VL(J,I,L)*TL(K,I,L)
  F12(I,L)=D14(I,L)*WLP(J,I,L)*TL(K,I,L)
  F13(I,L)=D16(I,L)*TL(J,I,L)*TL(K,I,L)
  F14(I,L)=-MXLP(J,I,L)*TL(K,I,L)
  F15(I,L)=D17(I,L)*TL(J,I,L)*TL(K,I,L)
  F16(I,L)=D13(I,L)*WL(J,I,L)*TL(K,I,L)
  F17(I,L)=-D15(I,L)*VL(J,I,L)*TL(K,I,L)
160 CONTINUE
DO 170 L=1,51
  F21(L)=-D22(L)*VP(J,L)*T(K,L)+OMEGAN*D25(L)*V(J,L)*T(K,L)
  F22(L)=D24(L)*WP(J,L)*T(K,L)
  F23(L)=D26(L)*T(J,L)*T(K,L)
  F24(L)=-MXP(J,L)*T(K,L)
  F25(L)=D27(L)*T(J,L)*T(K,L)
  F26(L)=D23(L)*W(J,L)*T(K,L)
  F27(L)=-D25(L)*V(J,L)*T(K,L)
170 CONTINUE
  A31(K,J)=INTG(F11,F12)
  A32(K,J)=INTG(F12,F22)
  A33(K,J)=INTG(F13,F23)
  A36(K,J)=INTG(F14,F24)
  B31(K,J)=INTG(F17,F27)
  B32(K,J)=INTG(F16,F26)
  B33(K,J)=INTG(F15,F25)
  A36(K,J)=A36(K,J)+T(K,1)*(MXL(J,1,11)+MXL(J,2,11)-MX(J,1))
  A35(K,J)=A35(K,J)-FD(1,1)*MYLP(J,1,11)*T(K,1)-FD(2,1)

```

1\*MYLP(J,2,11)\*T(K,1)

A32(K,J)=A32(K,J)+T(K,1)\*(D113(1,11)\*WLP(J,1,11)\*FD(1,1)  
1+D113(2,11)\*WLP(J,2,11)\*FD(2,1))

A33(K,J)=A33(K,J)+T(K,1)\*(FD(1,1)\*D12(1,11)\*T(J,1))  
1+FD(2,1)\*D12(2,11)\*T(J,2)+T(K,1)\*(FD(2,1)\*D14(2,11)\*  
2TL(J,1,11)+FD(2,2)\*D14(2,11)\*TL(J,2,11))

A34(K,J)=A34(K,J)+T(K,1)\*(FD(1,2)\*MZLP(J,1,11)+FD(2,2)  
1\*MZLP(J,2,11))

A31(K,J)=A31(K,J)\*T(K,1)\*(-FD(1,2)\*D113(1,11)\*VLP(J,1,11)  
1-FD(2,2)\*D113(2,11)\*VLP(J,2,11))

180 CONTINUE

DO 210 K=1,MODES

DO 210 J=1,MODES

DO 190 I=1,NPATH

DO 190 L=1,11

F11(I,L)=-OMEGAN\*D11(I,J)\*UL(J,I,L)\*UL(K,I,L)

F12(I,L)=-VXLP(J,I,L)\*UL(K,I,L)

F13(I,L)=D11(I,L)\*UL(J,I,L)\*UL(K,I,L)

190 CONTINUE

DO 200 L=1,51

F22(L)=-OMEGAN\*D21(L)\*U(J,L)\*U(K,L)

F22(L)=-VXP(J,L)\*U(K,L)

F23(L)=D21(L)\*U(J,L)\*U(K,J)

200 CONTINUE

A41(K,J)=INTG(F11,F21)

A42(K,J)=INTG(F12,F22)

B41(K,J)=INTG(F13,F23)

A42(K,J)=A42(K,J)+U(K,1)\*(VXL(J,1,11)+VXL(J,2,11)-VX(J,1))

210 CONTINUE

DO 240 K=1,MODES

DO 240 J=1,MODES

DO 220 I=1,NPATH

DO 220 L=1,11

F11(I,L)=UL(J,I,L)\*VXL(K,I,L)

F12(I,L)=-D18(I,J)\*VXL(J,I,L)\*VXL(K,I,L)

F13(I,L)=TL(J,I,L)\*MXL(K,I,L)

F14(I,L)=-D19(I,L)\*MXL(J,I,L)\*MXL(K,I,L)

220 CONTINUE

DO 230 L=1,51

F21(L)=U(J,L)\*VX(K,L)

F22(L)=-D28(L)\*VX(J,L)\*VX(K,L)

F23(L)=T(J,L)\*MX(K,L)

F24(L)=-D29(L)\*MX(J,L)\*MX(K,L)

230 CONTINUE

A51(K,J)=INTG(F11,F21)

A52(K,J)=INTG(F12,F22)

A61(K,J)=INTG(F13,F23)

A62(K,J)=INTG(F14,F24)

240 continue

DO 270 K=1,MODES

DO 270 J=1,MODES

DO 250 I=1,NPATH

DO 250 L=1,11

F11(I,L)=MZL(J,I,L)\*MZL(K,I,L)

F12(I,L)=D111(I,L)\*MYL(J,I,L)\*MZL(K,I,L)

F13(I,L)=-D110(I,L)\*MYL(J,I,L)\*MZL(K,I,L)

F14(I,L)=MYL(J,I,L)\*MYL(K,I,L)

F15(I,L)=-D112(I,L)\*MYL(J,I,L)\*MYL(K,I,L)

F16(I,L)=D112(I,L)\*MZL(J,I,L)\*MYL(K,I,L)

250 CONTINUE

DO 260 L=1,51

F21(L)=MZ(J,L)\*MZ(K,L)

F22(L)=D211(L)\*MY(J,L)\*MZ(K,L)

F23(L)=-D210(L)\*MY(J,L)\*MZ(K,L)

F24(L)=MY(J,L)\*MZ(K,L)

F25(L)=-D112(2,L)\*MY(J,L)\*MY(K,L)

F26(L)=D112(2,L)\*MY(J,L)\*MZ(K,L)

260 CONTINUE

A71(J,K)=INTG(F11,F21)

A72(J,K)=INTG(F13,F23)

A73(J,K)=INTG(F12,F22)

A81(J,K)=INTG(F14,F24)

A82(J,K)=INTG(F16,F26)

A83(J,K)=INTG(F15,F25)

270 CONTINUE

CALL SOLUTN(A52,MODES,-1,MODES)

CALL MATMUL(A52,A51,D1,5,5,5)

CALL SOLUTN(A62,MODES,-1,MODES)

CALL MATMUL(A62,A61,D2,5,5,5)

DO 280 I=1,MODES

DO 280 J=1,MODES

C(I,J)=A72(I,J)

C(I,J+MODES)=A73(I,J)

C(I+MODES,J)=A82(I,J)

C(I+MODES,J+MODES)=A83(I,J)

C1(I,J)=A71(I,J)

C1(I,J+MODES)=0.0

C1(I+MODES,J)=0.0

C1(I+MODES,J+MODES)=0.0

280 CONTINUE

N2=2\*MODES

CALL SOLUTN(C,N2,-1,N2)

CALL MATMUL(C,C1,C2,N2,N2,N2)

N3=3\*MODES

N4=4\*MODES

DO 285 I=1,N4

DO 285 J=1,N4

E1(I,J)=0.0

E2(I,J)=0.0

E3(I,J)=0.0

E4(I,J)=0.0

285 CONTINUE

DO 290 I=1,MODES

DO 290 J=1,MODES

E1(I,J+N2)=A11(I,J)

E1(I,J+N3)=A12(I,J)

E1(I+MODES,J+MODES)=A21(I,J)

E1(I+MODES,J+N3)=A22(I,J)

E1(I+N2,J+MODES)=A31(I,J)

E1(I+N2,J+N2)=A32(I,J)

E1(I+N2,J+N3)=A33(I,J)

E1(I+N3,J)=A41(I,J)

```
E2 (I, J) = A13 (I, J)
E2 (I, J+N2) = A14 (I, J)
E2 (I+MODES, J) = A23 (I, J)
E2 (I+MODES, J+MODES) = A24 (I, J)
E2 (I+N2, J+MODES) = A34 (I, J)
E2 (I+N2, J+N2) = A35 (I, J)
E2 (I+N2, J+N3) = A36 (I, J)
E2 (I+N3, J) = A42 (I, J)
E3 (I, J+N2) = B11 (I, J)
E3 (I, J+N3) = B12 (I, J)
E3 (I+MODES, J+MODES) = B21 (I, J)
E3 (I+MODES, J+N3) = B22 (I, J)
E3 (I+N2, J+N2) = B32 (I, J)
E3 (I+N2, J+MODES) = B31 (I, J)
E3 (I+N2, J+N3) = B33 (I, J)
E3 (I+N3, J) = B41 (I, J)
E4 (I, J) = -D1 (I, J)
E4 (I+N3, J+N3) = -D2 (I, J)
E4 (I+MODES, J+MODES) = C2 (I, J)
E4 (I+MODES, J+N2) = C2 (I, J+MODES)
E4 (I+N2, J+MODES) = C2 (I+MODES, J)
E4 (I+N2, J+N2) = C2 (I+MODES, J+MODES)
```

290 CONTINUE

```
CALL MATMUL (E2, E4, DS, N4, N4, N4)
CALL SOLUTN (E3, N4, -1, N4)
DO 300 I=1, N4
DO 300 J=1, N4
E2 (I, J) = E1 (I, J) + DS (I, J)
```



300 CONTINUE

CALL MATMUL(E3,E2,DS,N4,N4,N4)

RETURN

END

\*\*\*\*\*

FUNCTION INTG(F1,F2)

\*\*\*\*\*

-----

THIS FUNCTION INTEGRATES THE FUNCTION IN THE DOMAIN

-----

REAL INTG

DIMENSION F1(2,11),F2(51)

COMMON/XM1/ NPATH

COMMON/XM5/ HS1,HS2

H1=HS1/24.0

H2=HS2/24.0

SUM=0.0

DO 110 J=1,NPATH

SUM=SUM+(9.0\*F1(J,1)+19.0\*F1(J,2)-5.0\*F1(J,3)+F1(J,4))\*H1

DO 100 K=1,8

SUM=SUM+H1\*(-F1(J,K)+13.0\*F1(J,K+1)+13.0\*F1(J,K+2)-F1(J,K\_3))

100 CONTINUE

SUM=SUM+H1\*(F1(J,8)-5.0\*F1(J,9)+19.0\*F1(J,10)+9.0\*F1(J,11))

110 CONTINUE

SUM=SUM+(9.0\*F2(1)+19.0\*F2(2)-5.0\*F2(3)+F2(4))\*H2

DO 120 K=1,48

SUM=SUM+H2\*(-F2(K)+13.0\*F2(K+1)+13.0\*F2(K+2)-F2(K+3))

120 CONTINUE

INTG=SUM+H2\*(F2(48)-5.0\*F2(49)+19.0\*F2(50)+9.0\*F2(51))

RETURN

END

\*\*\*\*\*

SUBROUTINE INTPOL(N,A,H)

\*\*\*\*\*

-----

THIS SUBROUTINE INTERPOLATES FOR THE REQUIRED VALUES

-----

DIMENSION A(51),STA(51),TABLE(51,1),B(51)

COMMON/XM4/STA,NS

NN=N-1

A(N)=A(NS)

NM1=NS-1

DO 20 I=1,NM1

20 TABLE(I,1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))

XARG=H

DO 35 I=2,NN

DO 25 J=1,NS

IF (J.EQ.NS.OR.XARG.LE.STA(J)) GO TO 30

25 CONTINUE

30 MAX=J

IF (MAX.LE.2) MAX=2

ISUB=MAX-1

YEST=TABLE(ISUB,1)

B(I)=YEST\*(XARG-STA(ISUB))+A(ISUB)

35 XARG=XARG+H

```
DO 40 J=2,NN
```

```
40 A(J)=B(J)
```

```
RETURN
```

```
END
```

```
*****
```

```
SUBROUTINE MATMUL(A,B,C,L,M,N)
```

```
*****
```

```
-----
```

```
MATRIX MULTIPLICATION
```

```
-----
```

```
DIMENSION A(L,M),B(M,N),C(L,N)
```

```
DO 100 I=1,L
```

```
DO 100 J=1,N
```

```
C(I,J)=0.0
```

```
DO 100 K=1,M
```

```
C(I,J)=C(I,J)+A(I,M)*B(M,J)
```

```
100 CONTINUE
```

```
RETURN
```

```
END
```

```
*****
```

```
SUBROUTINE SOLUTN(A,N,INDIC,NRC)
```

```
*****
```

```
-----
```

```
SOLUTION OF SIMULTANEOUS EQUATIONS & INVERSE
```

```
-----
```

```
DIMENSION A(NRC,NRC),X1(100),IROW(100),JCOL(100),JORD(100),
```

```
1Y(100)
  IND=0
  MAX=N
  IF (INDIC.GE.0) MAX=N+1
  DETER=1.0
  DO 80 K=1,N
    KM1=K-1
    PIVOT=0.0
    DO 60 I=1,N
      DO 60 J=1,N
        IF (K.EQ.1) GO TO 55
        DO 50 ISCAN=1,KM1
          DO 50 JSCAN=1,KM1
            IF (I.EQ.IROW(ISCAN)) GO TO 60
            IF (J.EQ.JCOL(JSCAN)) GO TO 60
50      CONTINUE
55    IF (ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
        PIVOT=A(I,J)
        IROW(K)=I
        JCOL(K)=J
60    CONTINUE
        IF (ABS(PIVOT).GT.0.1E-15) GO TO 65
        DETER=0.0
        IND=1
        WRITE(50,61)
61    FORMAT('MATRIX IS ALGORITHMICALLY SINGULAR')
        RETURN
65    IROWK=IROW(K)
```

```
JCOLK=JCOL(K)
DETER=DETER*PIVOT
DO 70 J=1,MAX
70 A(IROWK,J)=A(IROWK,J)/PIVOT
A(IROWK,JCOLK)=1.0/PIVOT
DO 80 I=1,N
AIJCK=A(I,JCOLK)
IF (I.EQ.IROWK) GO TO 80
A(I,JCOLK)=-AIJCK/PIVOT
DO 75 J=1,MAX
75 IF (J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
80 CONTINUE
DO 85 I=1,N
IROWI=IROW(I)
JCOLI=JCOL(I)
JORD(IROWI)=JCOLI
IF (INDIC.GE.0) X1(JCOLI)=A(IROWI,MAX)
85 CONTINUE
INTCH=0
NM1=N-1
DO 90 I=1,NM1
IP1=I+1
DO 90 J=IP1,N
IF (JORD(J).GE.JORD(I)) GO TO 90
JTEMP=JORD(J)
JORD(J)=JORD(I)
JORD(I)=JTEMP
INTCH=INTCH+1
```

```
90 CONTINUE
  IF (INTCH/2*2.NE.INTCH) DETER=-DETER
  IF (INDIC.LE.0) GO TO 94
  RETURN

94 DO 100 J=1,N
  DO 95 I=1,N
    IROWI=IROW(I)
    JCOLI=JCOL(I)
    Y(JCOLI)=A(IROWI,J)

95 CONTINUE
  do 100 I=1,N
    A(I,J)=Y(I)

100 CONTINUE
  DO 110 I=1,N
  DO 105 J=1,N
    IROWJ=IROW(J)
    JCOLJ=JCOL(J)
    Y(IROWJ)=A(I,JCOLJ)

105 CONTINUE
  DO 110 J=1,N
    A(I,J)=Y(J)

110 CONTINUE
  RETURN
  END
```